Solution formulas for *n*-th degree equations

- Quadratic equation

The general form of a quadratic equation is:

$ax^2 + bx + c = 0$

The formula of the solution is given as

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Cubic equation

The general form is:

$ax^3 + bx^2 + cx + d = 0$

There are three roots for this equation:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -\frac{b}{3a} + \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} \\ -\frac{b}{3a} + \omega\sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \omega^2\sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} \\ -\frac{b}{3a} + \omega^2\sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \omega\sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} \end{pmatrix}$$

where

$$\omega = \frac{-1 \pm i\sqrt{3}}{2}$$

$$p = \frac{c}{a} - \frac{1}{3}\left(\frac{b}{a}\right)^2$$

$$q = \frac{d}{a} - \frac{1}{3}\frac{b}{a}\frac{c}{a} + \frac{2}{27}\left(\frac{b}{a}\right)^2$$

- Quartic equation

The equation is given as

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

The solutions are:

$$\begin{pmatrix} x_{1,2} \\ x_{3,4} \end{pmatrix} = \begin{pmatrix} -\frac{a}{4} + \frac{\sqrt{2s-p} \pm \sqrt{-(2s+p) - \frac{2q}{\sqrt{2s-p}}}}{2} \\ -\frac{a}{4} - \frac{\sqrt{2s-p} \pm \sqrt{-(2s+p) + \frac{2q}{\sqrt{2s-p}}}}{2} \end{pmatrix}$$

where

$$s = \frac{p}{6} + \sqrt[3]{-\frac{q'}{2} + \sqrt{\left(\frac{q'}{2}\right)^2 + \left(\frac{p'}{3}\right)^3}} + \sqrt[3]{-\frac{q'}{2} - \sqrt{\left(\frac{q'}{2}\right)^2 + \left(\frac{p'}{3}\right)^3}}$$

$$p = c - \frac{3b^2}{8}$$

$$q = d + \frac{b^3}{8} - \frac{bc}{2}$$

$$p' = -\frac{1}{12}\left(c - \frac{3b^2}{8}\right)^2 - \left(e - \frac{bd}{4} + \frac{bc}{16} - \frac{b^4}{64}\right)$$

$$q' = -\frac{1}{108}\left(c - \frac{3b^2}{8}\right)^3 - \frac{1}{8}\left(d + \frac{b^3}{8} - \frac{bc}{2}\right)^2 + \frac{1}{3}\left(c - \frac{3b^2}{8}\right)\left(e - \frac{bd}{4} + \frac{bc}{16} - \frac{b^4}{64}\right)$$