The formula for the special cases of Gauss integral:

$$\int_0^{\infty} x^p e^{-ax^q}\,dx = \frac{1}{qa^{(p+1)/q}}\Gamma\left(\frac{p+1}{q}\right) \qquad (1)$$

(Proof)
The gamma function is given as follows:

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt \qquad (x > 0)$$

Let $ax^q = t$; then, we have $x = \frac{1}{a^{1/q}} t^{1/q}$ and $dx = \frac{1}{a^{1/q}}\frac{1}{q} t^{(1/q)-1} dt$. Substitute them into the above. Thus, we have

$$\int_0^{\infty} \left(\frac{t^{1/q}}{a^{1/q}}\right)^p e^{-t} \frac{1}{a^{1/q}}\frac{1}{q} t^{(1/q)-1} dt = \frac{1}{qa^{(p+1)/q}} \int_0^{\infty} e^{-t} t^{(1/q)-1} dt = \frac{1}{qa^{(p+1)/q}}\Gamma\left(\frac{p+1}{q}\right).$$

Q.E.D.

The gamma function gives the following values:

$$\Gamma(1) = 1 \qquad (2)$$

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi} \qquad (3)$$

$$\Gamma(n+1) = n! \text{ with } n = 1,2,3\ldots \qquad (4)$$

$$\Gamma(x) = (x-1)\Gamma(x-1) \text{ with } x > 1 \qquad (5)$$

e.g $\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{1}{2}\sqrt{\pi}$

(Example)

$$\int_0^{\infty} x^4 e^{-ax^2} dx = ?$$

From the above formula, (1), we can let $p = 4$ and $q = 2$. Therefore,

$$\int_0^{\infty} x^4 e^{-ax^2} dx = \frac{1}{2a^{5/2}}\Gamma\left(\frac{5}{2}\right)$$

Then, use (5).

$$= \frac{1}{2a^{5/2}}\frac{3}{2}\Gamma\left(\frac{3}{2}\right)$$

$$= \frac{1}{2a^{5/2}}\frac{3}{2}\cdot\frac{1}{2}\Gamma\left(\frac{1}{2}\right)$$

$$= \frac{3}{8a^{5/2}}\sqrt{\pi}$$