How to derive a wave equation

There is a time-dependent first order differential equation:

$$\frac{dv_x(t)}{dt} = \omega v_y(t) \qquad (1)$$

Note that the time-derivative of v_x is proportional to v_y. Take another derivative of equation (1).

$$\frac{d^2v_x(t)}{dt^2} = \omega\frac{dv_y(t)}{dt} \qquad (1)'$$

Here is another equation that is the pair of equation (1).

$$\frac{dv_y(t)}{dt} = -\omega v_x(t) \qquad (2)$$

Solve for dv_y/dt in equation (1)'.

$$\frac{dv_y(t)}{dt} = \frac{1}{\omega}\frac{d^2v_x(t)}{dt^2} \qquad (1)''$$

Plug (1)" into (2).

$$\frac{1}{\omega}\frac{d^2v_x(t)}{dt^2} = -\omega v_x(t) \qquad (2)'$$

Therefore, we have the wave equation on x-axis:

$$\frac{d^2v_x(t)}{dt^2} = -\omega^2 v_x(t) \qquad (3)$$

Try this: Derive the wave equation on y-axis by using (1) and (2).