Chiral Symmetry and the Nucleon-Nucleon Interaction: Developing a Chiral NN Potential in Configuration Space

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Abstract

We develop a nucleon-nucleon (NN) potential based upon chiral perturbation theory at next-to-next-to-leading order (N^{2}LO). We represent this potential in configuration space (r-space) such that it can be applied by nuclear few- and many-body theoreticians who work in r-space. The potential consists of one-pion exchange, two-pion exchange up to N^{2}LO, and eight contact interactions up to power Q^2 (where Q denotes a generic momentum).

We apply the potential to NN scattering and the deuteron (two-nucleon bound state). The eight parameters of the contact terms are adjusted such that the two-nucleon data are described as good as possible. The phase shifts of neutron-proton scattering are reproduced in general well, with the exception of two D- and F-waves. The predicted deuteron properties agree with the empirical data.

Even though this potential is only semi-quantitative, it represents considerable progress towards a quantitative chiral r-space potential.

Introduction

The atomic nucleus was discovered by Rutherford in the year of 1911 by demonstrating that large-angle alpha-particle scattering can only be described in terms of a positively charged nucleus with a very small radius [1]. Then, Thomson studied the nuclear mass and discovered the existence of isotopes [2]. The first nuclear models employed protons and electrons with only electrostatic forces to construct a nucleus. In 1932, the neutron was discovered by Chadwick [3]. This suggested that the neutron and the proton were the basic constituents of nuclei. Furthermore, one had to consider the existence of a new force between neutrons and protons to bind the nucleus, which was called the nuclear force or the strong force.

In the next phase, Wigner concluded that the nuclear force had to be of short range and strong within that range [4]. Heisenberg [5] and Majorana [6] theoretically discussed the new forces and introduced the concept of "exchange forces" to explain nuclear saturation. Around this period, experiment also made big progress, such as measuring the binding energy of the deuteron [7] and conducting proton-proton scattering experiments [8]. Heisenberg already indicated that the neutron and proton can be recognized as two states of the same particle [5]. The concept of isospin was introduced by Cassen and Condon in 1936 [9].

The first fundamental idea for the origin of the nuclear force was created by Yukawa [10] in 1935. He assumed that nucleons interact via the exchange of massive scalar particles. This creates a potential, which is proportional to $\exp(-\mu r)/r$. The exponential contains the meson mass μ, and r is the distance between the two nucleons. There have been many modifications of the meson-exchange theory of nuclear forces over the past sixty years. However, the basic concept, created by Yukawa, proved to be right.

The first modifications of Yukawa's theory were extensions of his model to pseudoscalar and pseudovector particles by Proca [11] and Kemmer [12]. The quadrupole moment and the magnetic moment of the deuteron were measured by Kellog, Rabi, Ramsey, and Zacharias [13] [14] in 1939. Møller, Rosenfeld [15], and Schwinger [16] derived a tensor force giving rise to the quadrupole moment by employing both pseudoscalar and vector fields. In 1946, Pauli predicted the existence of an isovector pseudoscalar meson because the exchange of a particle with these quantum numbers could explain the sign of the quadrupole moment [17]. The predicted particle, which was called pion or π-meson, was observed experimentally a few years later [18]. It was also recognized by Breit [19] [20] and Rosenfeld [21] that vector and scalar fields create a spin-orbit force.

In 1951, Taketani, Nakamura, and Sasaki introduced the concept of subdividing the nuclear force into three regions [22]. They classified a long-range ($r \geq 2$ fm), an intermediate range (1 fm $\leq r \leq$ 2 fm), and a short range ($r \leq 1$ fm). For the longest-range part, the one pion exchange (OPE) is dominant because of the small mass of the pion. For the intermediate-range, two pion exchange (TPE) is important. In the core region, various processes contribute to the interaction, such as multipion, heavy meson and quark-gluon exchanges.

In the 1950's, OPE became well established for the long-range part of the nuclear interaction. Besides that, theoreticians paid considerable attention to the TPE contribution. Taketani, Machida, and Onuma evaluated an S matrix directly from meson field theory [23]. Brueckner and Watson derived a potential based on an expansion in the particle number [24]. However, the pair terms had to be excluded in accordance with the experimental result, and the suppression of virtual pairs was assumed as a general rule ('pair supression') [25]. This led ultimately to the concept of chiral symmetry [26] [27] [28]. Furthermore, it turned out that the spin-orbit force derived from TPE was much weaker than experimentally needed.

In 1960, Breit and others revived the old idea of vector-meson exchange. Soon after, the vector mesons ρ [29] and ω [30] were discovered. ρ and ω are 2π and 3π resonances, respectively. The one-boson-exchange (OBE) model was developed. This model provided a remarkable quantitative agreement with the existing scattering data using only few parameters [31] [32] [33].

To further develop the meson theory of nuclear force, dispersion relations and field theoretical approaches were used. The most elaborated work applying dispersion relations was done by the Stony Brook [34] and the Paris [35] groups. Lomon et.

al. [36] used the field-theoretic approach and calculated the Feynman diagrams of the 2π-exchange contributions to the NN interaction.

The Bonn group developed one of most elaborate models within the meson-exchange concept. Holinde, Machleidt, and Elster calculated two- and some of the three- and four-pion exchange diagrams and extended the model to take the effects of virtual isobar excitations into account [37]. The Bonn-potential accurately agrees with the NN scattering data. Some interesting attempts were performed by the Jülich group to incorporate also correlated meson exchanges (like $\pi\pi$, $\pi\rho$) [38] [39] [40].

The Nijmegen group constructed OBE potentials [41], and performed a partial-wave analysis (PWA) of pp and np scattering data [42].

There is another high-quality NN force that is called Charge-Dependent Bonn (CD-Bonn) potential based on the relativistic OBE model [43]. It includes nonlocality which increases the triton binding energy.

The Argonne group used a more phenomenological approach. For the long range, they use one-pion exchange, and for the short-range phenomenology. This NN potential includes charge-dependence, and charge-asymmetry [44], as does the CD-Bonn.

There are off-shell differences between different NN potentials. Because of the off-shell ambiguity, the binding energy of the triton varies remarkably when calculations are executed with different two-nucleon forces. The missing binding energy is, generally, attributed to three-nucleon forces. There is a good review article about three-nucleon physics [45].

There are several models for the three-nucleon force. The Fujita-Miyazawa [46] and the Tucson-Melbourne [47] forces are based upon two-pion exchange with one intermediate Δ excitation. The two-pion exchange interaction is the longest range part of the 3NF. The Brazil group includes $\pi-\rho$ and $\rho-\rho$ exchange, additionally [48]. The Urbana-Argonne group worked out a phenomenological 3NF [49]. The Tucson-Melbourne force has also been extended to include the $\pi-\rho$ and $\rho-\rho$ exchanges [50]. Even though one can adjust the parameters for the 3NF to reproduce the triton binding energy, the analyzing power A_y in elastic neutron-deuteron (nd) scattering at low energy could not be reproduced. Including Tucson-Melbourne model does not even provide a significant improvement. This is still an unsolved problem [51].

Despite a very successful description of most of the experimental data, the meson-exchange based NN potentials are essentially phenomenological models because mesons are not fundamental particles. Moreover, the models use a fictitious σ boson. It is needed to produce the strong attraction in the central part of the potential. However, the existence of such a meson is controversial. In addition, form factors are applied, which are not well-defined in quantum field theory. Those are *ad hoc* quantities applied at each vertex in order to correct the large momentum behavior of the potential.[1]

There may also be conceptual problems with the meson exchange picture. Simply put, assuming that the hadrons are hard spheres, the typical size of a light meson is about 0.5 fm, which is almost the charge radius of a proton. The meson will not be able to fit between nucleons. This may suggest that the model is adequate only for distance of more than 2 fm.

A more fundamental approach to the nuclear force would start from QCD. QCD is believed to be the fundamental theory of strong interactions. It is an $SU(3)_{\text{color}}$ gauge field theory, using quarks and gluons as the degrees of freedom. However, QCD is non-perturbative in the low energy regime, such as nuclear physics. Therefore, QCD-inspired models have been constructed. For example, a six-quark system has been considered for the deuteron. This result actually matches up to the quantum numbers of the nucleons, and could explain several features of low-energy nuclear physics [52].

The Moscow (Russia) NN potential [53] is a hybrid model. The long-range interaction is described in terms of meson-exchanges. At short-range, one-gluon exchange and confinement interactions are considered. Several QCD-inspired quantitative quark models have been constructed for the nuclear force. More details can be found in the review article [54].

A QCD-based approach that uses the concept of an effective field theory (EFT) was suggested some 20 years ago by Weinberg [55]. One writes down the most gen-

[1] Only in the Bochum (Ruhr-) potential, the form factors are generated by solving the corresponding integral equations. [65]

eral Lagrangian that observes the (broken) chiral symmetry of QCD and all the other symmetries, and uses pions and nucleons as the effective degrees of freedom. This is called chiral effective field theory. Although applying the Lagrangian to NN scattering generates an infinite number of Feynman diagrams, Weinberg gave an idea of the systematic expansion of the amplitude in terms of $(Q/\Lambda_\chi)^\nu$, where Q denotes a momentum or pion mass, $\Lambda_\chi \approx 1$GeV is the scale of chiral symmetry breaking, and $\nu \geq 0$ is the order of the expansion. This approach, known as Chiral Perturbation Theory (CHPT), is quite successful in the $\pi\pi$ and πN sectors. Weinberg also looked into the problem of the NN interaction [56] [57] [58]. He proposed to use CHPT to calculate the kernel of the Lippmann-Schwinger (LS) equation that describes NN scattering. This kernel is identified with the NN potential. Or, in other words: the NN potential is calculated perturbatively. The NN amplitude is calculated non-perturbatively (solution of LS equation). The existence of large scattering lengths and of a shallow bound state (deuteron) make the non-perturbative part of the calculation necessary.

Following this philosophy, the Texas-Seattle group obtained an energy dependent two-nucleon potential in r-space at next-to-next-to-leading order (N^2LO) [59] [60] [61].

CHPT applied to the NN system has also been used by several other researchers. Kaiser, Brockmann, and Weise presented the first model-independent prediction for the NN amplitudes of peripheral partial waves at N^2LO [62].

The purpose of this thesis is to construct an NN potential based upon effective field theory in configuration space. Chiral NN potentials in momentum space have been constructed by other researchers, such as Epelbaum, Glöckle, and Meißner [63], and Entem and Machleidt [64]. However, many microscopic nuclear structure calculations are conducted in configuration space (r-space). For this research, an NN potential represented in r-space is needed. The only existing chiral r-space potential is Ref. [60]. However, this potential is not quantitative, and therefore not suitable for nuclear structure calculations. Thus, there is a need for a quantitative chiral r-space NN potential. It is the goal of this thesis to do the first step for the construction of such a potential.

Conclusions

In this thesis, we have developed a quantitative nucleon-nucleon potential in configuration space applying chiral perturbation theory up to next-to-next-to-leading order. We regularized the potential at short distances and applied it to NN scattering and the bound state (deuteron) problem. We here summarize the results of our research.

Chiral perturbation theory has turned out to be a useful approach to deal with the nucleon-nucleon interaction. For the π and 2π part of the potential, we follow Ref. [62]. The potential consists of central, spin-spin, spin-orbit, and tensor terms for both isoscalar and isovector part of the potential. We use the contact terms up to next-to leading order. From the second order contacts, we include the q^2 terms, k^2 terms, spin-orbit term, and tensor term in $\vec{q}$. We omit the tensor term in $\vec{k}$.

The low energy constants c_1, c_3, and c_4 involved in the 2π contribution are adjusted to the phase shifts of peripheral partial waves, which are reproduced very well.

For the lower partial waves, the inclusion of the contact terms is crucial. The eight contact parameters can be chosen such as to reproduce the S and P waves well. Some persistent discrepancies remain in ${}^3D_2$, ${}^3D_3$, ${}^3F_3$, and ${}^3F_4$. However, overall our fit of the np phase shifts is a substantial improvement over Ref. [60].

The deuteron properties are also reproduced well by our potential.

We also emphasize that a model derived from the local potential is necessary for nuclear theoreticians who are studying few- and many-body systems in configuration space. This work will be a good start to develop a higher precision chiral NN r-space potential, and it will contribute to nuclear r-space theoreticians.

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