Chiral Symmetry and the Nucleon-Nucleon Interaction: Developing a Chiral NN Potential in Configuration Space

Hiro Shimoyama

Outline

1. Motivation

2. Introduction

QCD and Chiral Effective Field Theory

3. Spontaneous Symmetry Breaking

Explicit and Spontaneous Symmetry Breaking

4. The Chiral Lagrangian

5. The NN Potential

OPE, TPE, and Contacts

6. Results

Phase Shifts and Deuteron

7. Conclusions

1. Motivation

- One important goal of theoretical nuclear physicists is to understand atomic nuclei from nucleons and the forces between them ("microscopic nuclear structure").

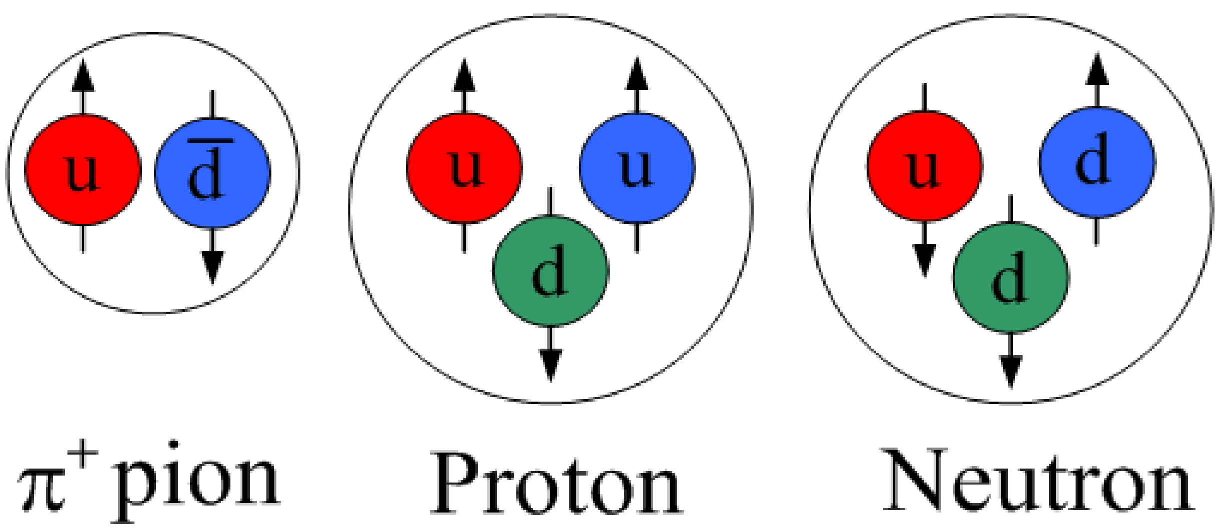
- A crucial ingredient for the above approach is a nucleon-nucleon (NN) potential.

- Therefore, we will develop an NN potential (in configuration space) for application in nuclear structure.

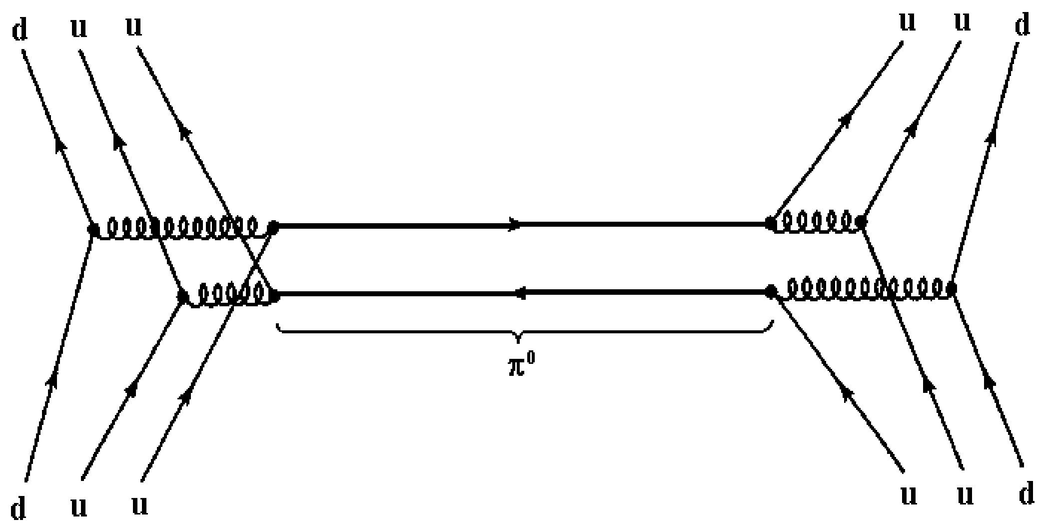
2. Introduction

- Quantum Chromodynamics is the fundamental theory of strong interactions. (quark-quark)

- Hadrons are made from quarks.



- Nucleon-Nucleon interaction is based on quark-quark interactions (QCD).



However, ...

Nuclear physics is "Low-Energy Physics."
So, the coupling constant is large.
Non-perturbative. Problem!

$\Downarrow$

The “solution”:

An “Effective Field Theory (EFT)”

using effective degrees of freedom: pions and nucleons

$\Downarrow$

Since this EFT is the low energy version of QCD; therefore, it must have the same symmetries as QCD; particularly,

chiral symmetry.

$\Downarrow$

Chiral Effective Field Theory

3. Spontaneous Symmetry Breaking

- The QCD Lagrangian

$$\mathcal{L}_{QCD} = \bar{q}i \not{D} q - \bar{q}\mathcal{M}q - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

with $D_\mu \equiv \left(\partial_\mu + igT^a A^a_\mu\right)$

The masses of the quarks break chiral symmetry explicitly.

$\Downarrow$

However, the "up" and "down" quarks are very light.

($m_u \approx 5$MeV and $m_d \approx 9$MeV $<<$ hadronic scale of ≈ 1GeV)

$\Downarrow$

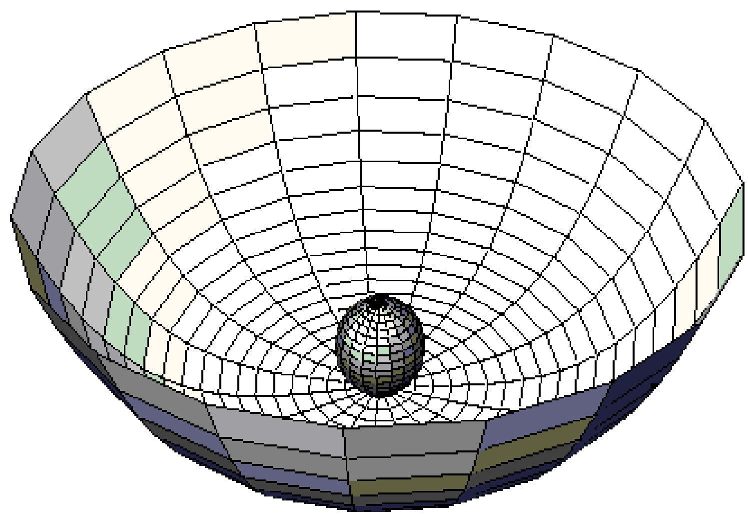
Therefore, the masses are almost negligible.

$\Downarrow$

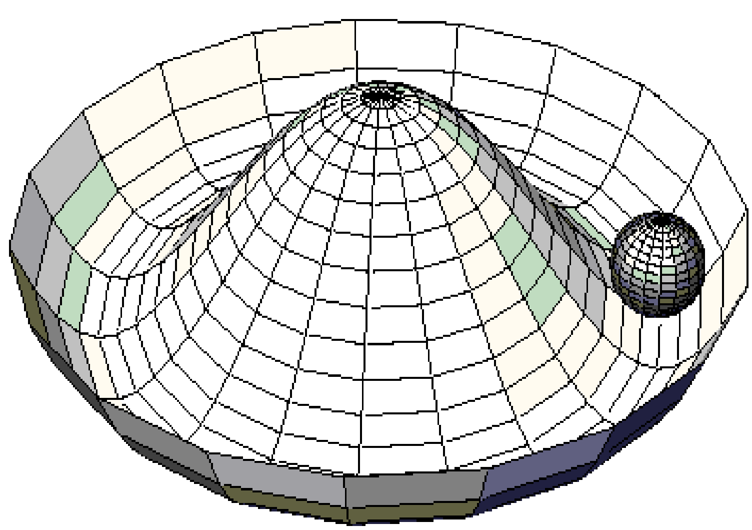
QCD in the light quark sector is approximately Chiral-Symmetric.

• Spontaneous symmetry breaking

Chiral symmetry is spontaneously broken in the ground state.



$\Downarrow$



Chiral symmetry would imply that the hadron spectrum has parity doublets.

$\Downarrow$

However, those are not observed.

$\Downarrow$

The symmetry is <u>spontaneously</u> broken.

4. The Chiral Lagrangian

<u>From Weinberg's Theorem:</u>

The effective Lagrangian involving the pion fields has to contain all terms consistent with the spontaneously broken chiral symmetry, etc. $\Downarrow$

$$\mathcal{L}^{(1)}_{\pi N} = \bar{N}[i\partial_0 - \frac{1}{4f_\pi^2}\vec{\tau}\cdot(\vec{\pi}\times\partial_0\vec{\pi}) - \frac{g_A}{2f_\pi}\vec{\tau}\cdot(\vec{\sigma}\cdot\vec{\nabla})\vec{\pi}]N + ...$$

$$\begin{aligned}\mathcal{L}^{(2)}_{\pi N} &= \bar{N}[\frac{1}{2M}\vec{D}\cdot\vec{D} + i\frac{g_A}{4M}\{\vec{\sigma}\cdot\vec{D},\cdot u_0\} \\ &+ 2c_1 m_\pi^2(U+U^\dagger) + \left(c_2 - \frac{g_A^2}{8M}\right)u_0^2 \\ &+ c_3 u_\mu\cdot u^\mu + i\frac{1}{2}\left(c_4 + \frac{1}{4M}\right)\vec{\sigma}\cdot(\vec{u}\times\vec{u})]N.\end{aligned}$$

with $U = e^{i\vec{\tau}\cdot\vec{\pi}/f_\pi}$ and $u_\mu = iu^\dagger\nabla_\mu U u^\dagger$

5. The *NN* Potential

1. Start from the Lagrangian for the pion-nucleon interactions, and using Feynman rules, we evaluate the NN diagrams, up to a certain order of **chiral perturbation theory.**

2. Identify the set of diagrams with the *NN* potential.

• Diagrams are of order $(Q/\Lambda)^\nu$ with

$$\nu = \mathbf{2} \times \mathbf{loops} + \sum_{\text{vertices}\,j} \left(\mathbf{d_j} + \frac{\mathbf{n_j}}{\mathbf{2}} - \mathbf{2} \right)$$

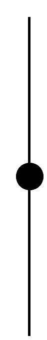
$\mathbf{d_j}$ ≡ power of the vertex

$$\cdot \rightarrow d_j = 1, \qquad \circ \rightarrow d_j = 1$$

$$\bullet \rightarrow d_j = 2, \qquad \odot \rightarrow d_j = 2$$

$\mathbf{n_j}$ ≡ number of nucleon lines at a vertex

$$n_j = 2$$



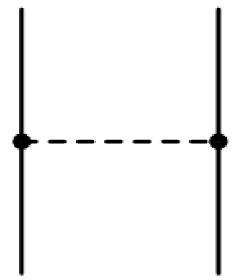
• General form of NN amplitude

$$
\begin{aligned}
\boldsymbol{V} = \; & \boldsymbol{V_C} + \vec{\boldsymbol{\tau}}_1 \cdot \vec{\boldsymbol{\tau}}_2 \, \boldsymbol{W_C} \\
& + [\, \boldsymbol{V_S} + \vec{\boldsymbol{\tau}}_1 \cdot \vec{\boldsymbol{\tau}}_2 \, \boldsymbol{W_S} \,] \, \vec{\boldsymbol{\sigma}}_1 \cdot \vec{\boldsymbol{\sigma}}_2 \\
& + [\, \boldsymbol{V_T} + \vec{\boldsymbol{\tau}}_1 \cdot \vec{\boldsymbol{\tau}}_2 \, \boldsymbol{W_T} \,] \, \boldsymbol{S_{12}} \\
& + [\, \boldsymbol{V_{LS}} + \vec{\boldsymbol{\tau}}_1 \cdot \vec{\boldsymbol{\tau}}_2 \, \boldsymbol{W_{LS}}] \, \vec{\boldsymbol{L}} \cdot \vec{\boldsymbol{S}}
\end{aligned}
$$

where

$$
\boldsymbol{S_{12}} = 3 \frac{(\vec{\boldsymbol{\sigma}}_1 \cdot \vec{\boldsymbol{r}})(\vec{\boldsymbol{\sigma}}_2 \cdot \vec{\boldsymbol{r}})}{\boldsymbol{r}^2} - \vec{\boldsymbol{\sigma}}_1 \cdot \vec{\boldsymbol{\sigma}}_2
$$

Q^0
(LO)

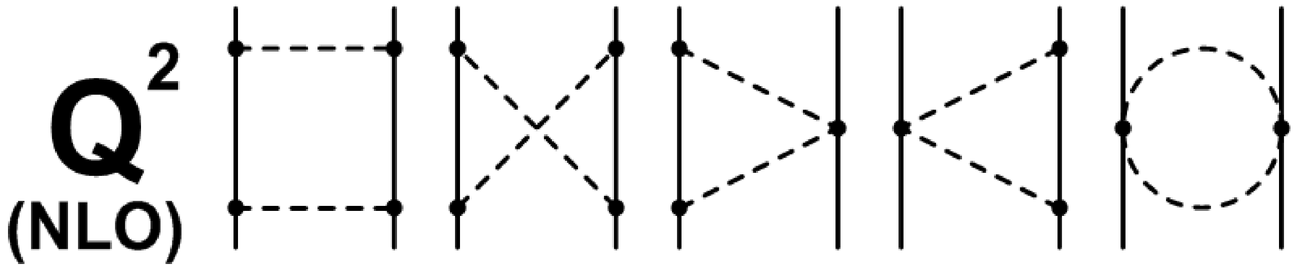


Lowest Order/Leading Order (LO) Contribution:

Static, non-relativistic One-Pion-Exchange (OPE)

$$W_S = \frac{g_A^2 m_\pi^2}{48\pi f_\pi^2} \frac{e^{-x}}{r}$$

$$W_T = \frac{g_A^2}{48\pi f_\pi^2} \frac{e^{-x}}{r^3} (3 + 3x + x^2)$$



Next-to-Leading Order (NLO)

"leading order 2π exchange"

$$
\begin{aligned}
W_C &= \frac{m_\pi}{128\pi^3 f_\pi^4 r^4}[\{1 + 2g_A^2(5 + 2x^2) - g_A^4(23 + 12x^2)\}K_1(2x) \\
&\quad + x\{1 + 10g_A^2 - g_A^4(23 + 4x^2)\}K_0(2x)] \\
V_S &= \frac{g_A^4 m_\pi}{32\pi^3 f_\pi^4 r^4}\left\{3xK_0(2x) + (3x + 2x^2)K_1(2x)\right\} \\
V_T &= \frac{g_A^4 m_\pi}{128\pi^3 f_\pi^4 r^4}\left\{-12xK_0(2x) - (15 + 4x^2)K_1(2x)\right\}
\end{aligned}
$$

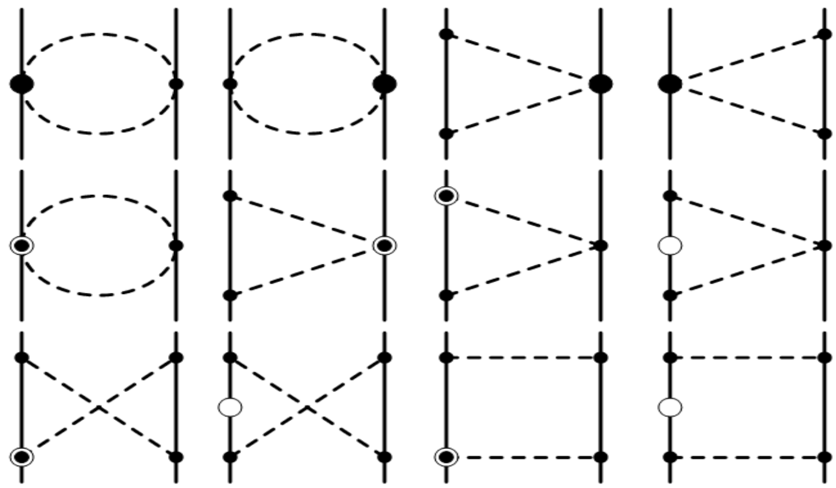
where

$$K_0(x)$$

and

$$K_1(x)$$

are the modified Bessel functions.



Next-to-Next-to-Leading Order

(NNLO, N^{2}LO)

"sub-leading 2π exchange"

$$
\begin{aligned}
V_C &= \frac{3g_A^2}{32\pi^2 f_\pi^4}\frac{e^{-2x}}{r^6}\left\{\left(2c_1+\frac{3g_A^2}{16M_N}\right)x^2(1+x)^2+\frac{g_A^2x^5}{32M_N}\right. \\
&+ \left.\left(c_3+\frac{3g_A^2}{16M_N}\right)(6+12x+10x^2+4x^3+x^4)\right\} \\
W_C &= \frac{g_A^2 m_\pi}{512\pi^2 M_N f_\pi^4}\frac{e^{-2x}}{r^5} \\
&\times \left\{2(3g_A^2-2)(6x^{-1}+12+10x+4x^2+x^3)+g_A^2x(2+4x+2x^2+3x^3)\right\} \\
V_S &= \frac{-3g_A^4 m_\pi}{512\pi^2 f_A^4 M_N}\frac{e^{-2x}}{r^5}(6x^{-1}+12+11x+6x^2+2x^3) \\
V_T &= \frac{3g_A^4 m_\pi}{1024\pi^2 f_A^4 M_N}\frac{e^{-2x}}{r^5}(12x^{-1}+24+20x+9x^2+2x^3) \\
W_S &= \frac{g_A^2}{48\pi^2 f_\pi^4}\frac{e^{-2x}}{r^6} \\
&\times \left\{\left(c_4+\frac{1}{4M_N}\right)(1+x)(3+3x+2x^2)-\frac{g_A^2}{16M_N}(18+36x+31x^2+14x^3+2x^4)\right\} \\
W_T &= \frac{g_A^2}{48\pi^2 f_\pi^4}\frac{e^{-2x}}{r^6} \\
&\times \left\{-\left(c_4+\frac{1}{4M_N}\right)(1+x)(3+3x+x^2)+\frac{g_A^2}{32M_N}(36+72x+52x^2+17x^3+2x^4)\right\} \\
V_{LS} &= -\frac{3g_A^4}{64\pi^2 M_N f_\pi^4}\frac{e^{-2x}}{r^6}(1+x)(2+2x+x^2) \\
W_{LS} &= \frac{g_A^2(g_A^2-1)}{32\pi^2 M_N f_\pi^4}\frac{e^{-2x}}{r^6}(1+x)^2
\end{aligned}
$$

• Have we gotten all the potential?

$$\Downarrow$$

Contact Interactions
The short-range interaction

• Contact terms in momentum space

Zero-th order

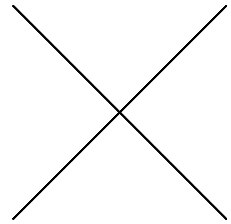
$$V^{(0)}(\vec{q},\vec{k}) = \frac{1}{(2\pi)^3}\left[C_S + C_T(\vec{\sigma_i}\cdot\vec{\sigma_j})\right] f_i(q^2),$$

with $f_i(q^2) = e^{-\left(\frac{q}{\Lambda}\right)^2}$.

Second order

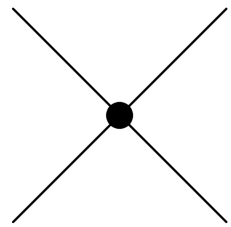
$$\begin{aligned} V^{(2)}(\vec{q},\vec{k}) &= \frac{1}{(2\pi)^3}[C_1 q^2 + C_2 k^2 \\ &+ (C_3 q^2 + C_4 k^2)\vec{\sigma}_i\cdot\vec{\sigma}_j \\ &+ C_5\left(-i\vec{S}\cdot(\vec{q}\times\vec{k})\right) \\ &+ C_6(\vec{\sigma}_i\cdot\vec{q})(\vec{\sigma}_j\cdot\vec{q}) \\ &+ C_7(\vec{\sigma}_i\cdot\vec{k})(\vec{\sigma}_j\cdot\vec{k})]f_i(q^2). \end{aligned}$$

Contact Terms (in r-space)



Zero-th Order

$$V^{(0)}(\vec{r}) = [C_S + C_T(\vec{\sigma}_i \cdot \vec{\sigma}_j)]\,\tilde{f}_g(r)$$
$$\text{with} \quad \tilde{f}_g(r) = \frac{\Lambda^3}{8\pi^{3/2}} e^{-\frac{\Lambda^2 r^2}{4}}$$



Second Order

♢ q^2 term:

$$V_1^{(2)}(\vec{r}) = [C_1 + C_3(\vec{\sigma}_i \cdot \vec{\sigma}_j)]\Lambda^2 \left(\tfrac{3}{2} - \tfrac{\Lambda^2 r^2}{4}\right) \tilde{f}_g(r)$$

♢ k^2 term:

$$V_2^{(2)}(\vec{r}) = [C_2 + C_4(\vec{\sigma}_i \cdot \vec{\sigma}_j)]\Big\{\tfrac{\Lambda^2}{4}\left(\tfrac{\Lambda^2 r^2}{4} - \tfrac{3}{2}\right)\tilde{f}_g(r) - \tfrac{1}{2}[\vec{\nabla}_r^2 \tilde{f}_g(r) + \tilde{f}_g(r)\vec{\nabla}_r^2]\Big\}$$

♢ Spin-Orbit term:

$$V_3^{(2)}(\vec{r}) = C_5 \tfrac{\Lambda^2}{2} \tilde{f}_g(r) \vec{L} \cdot \vec{S}$$

♢ Tensor term:

$$V_4^{(2)}(\vec{r}) = C_6 \tfrac{\Lambda^2}{3} \tilde{f}_g(r) \left[\left(\tfrac{3}{2} - \tfrac{\Lambda^2 r^2}{4}\right)(\vec{\sigma}_i \cdot \vec{\sigma}_j) - \tfrac{\Lambda^2 r^2}{4} S_{ij}\right]$$

- Define the potential:

$$\mathbf{V} = \mathbf{OPE} + \mathbf{2}\pi + \mathbf{Contacts}$$

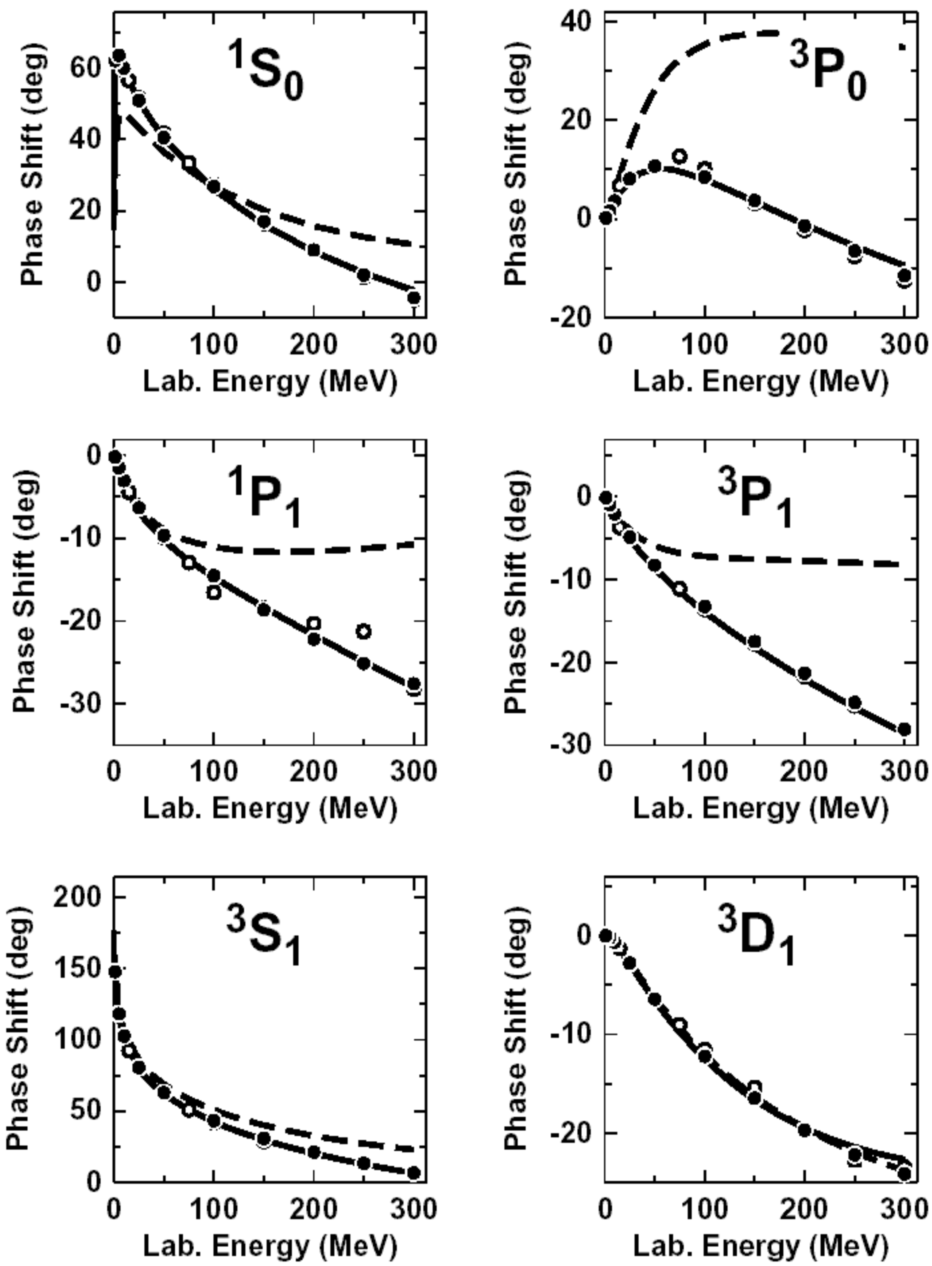
- Adjust parameters:

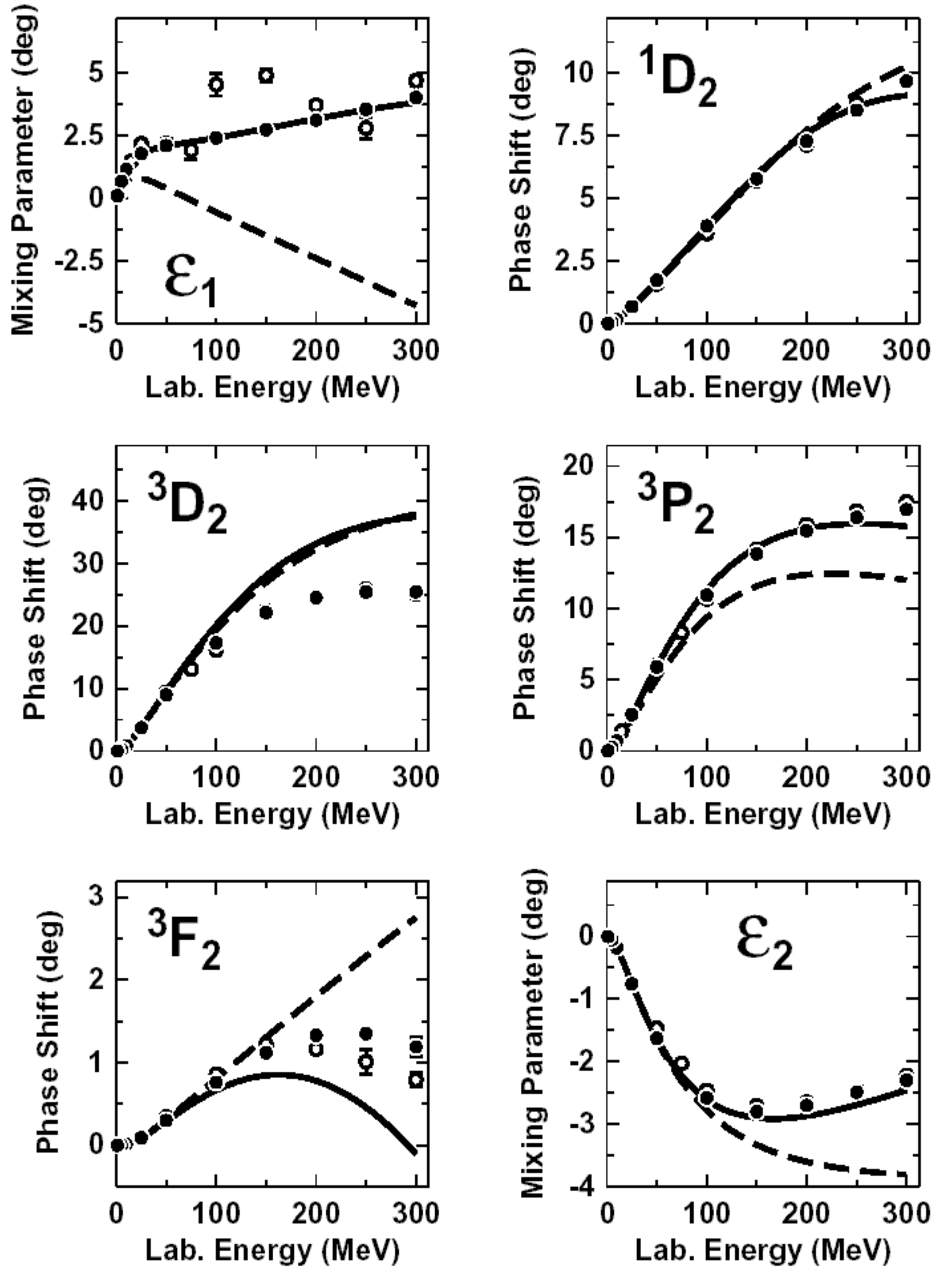
1. Fix the Low Energy Constants in the $\pi + 2\pi$ potential using the phase shifts of peripheral waves.

2. Fix the eight parameters of the contacts using phase shifts of the lower partial waves.

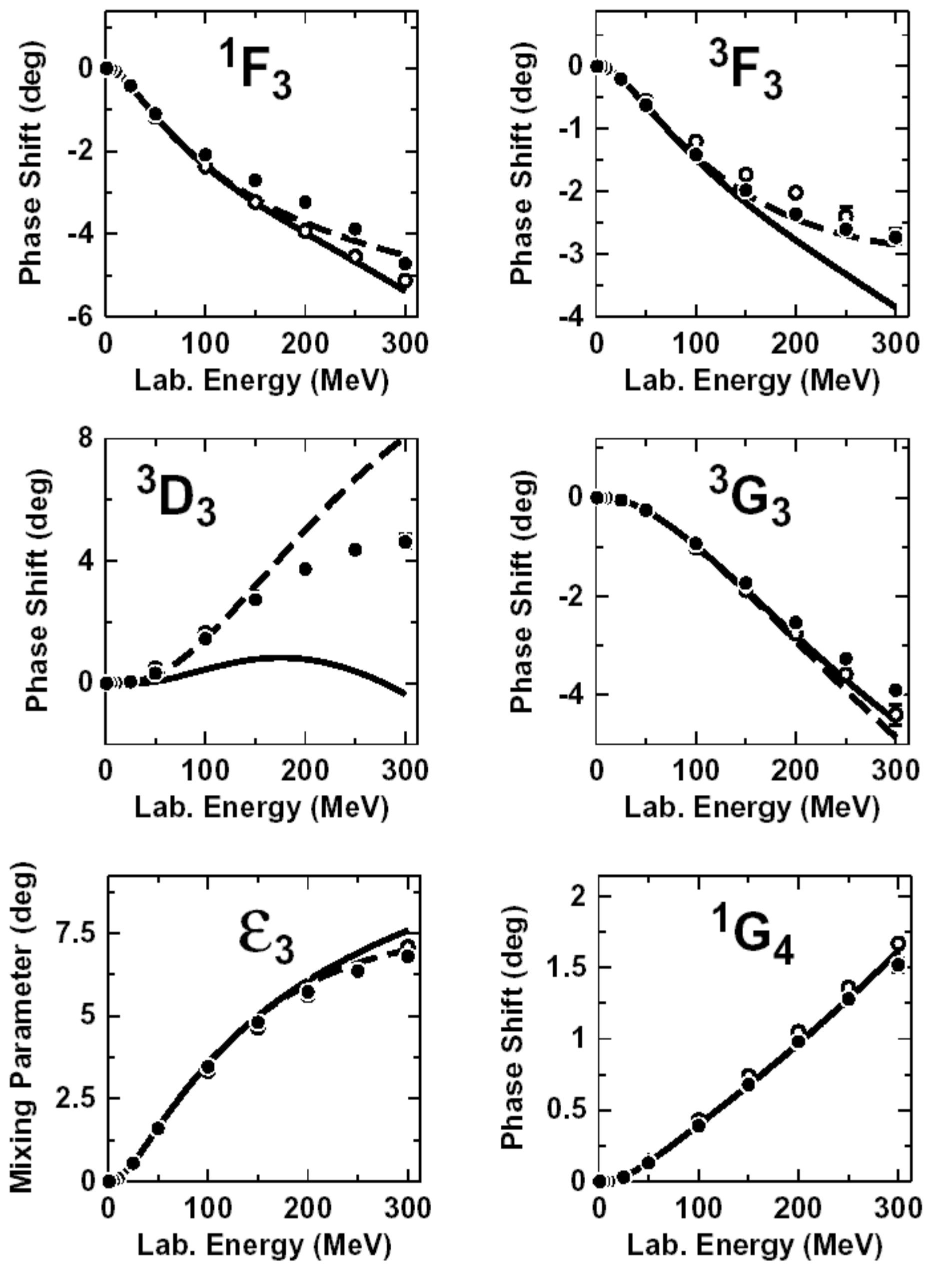
6. Results

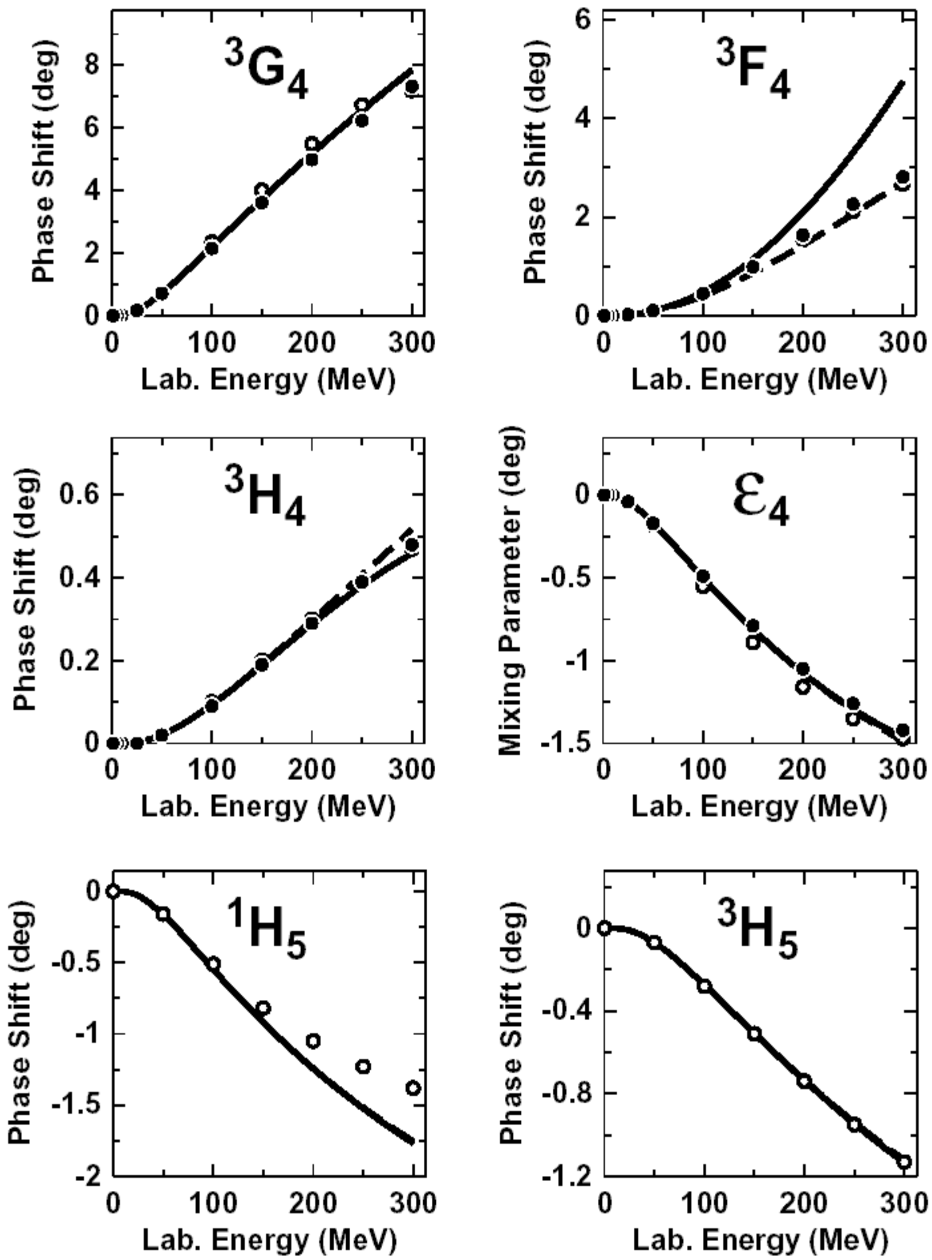
- Phase shifts



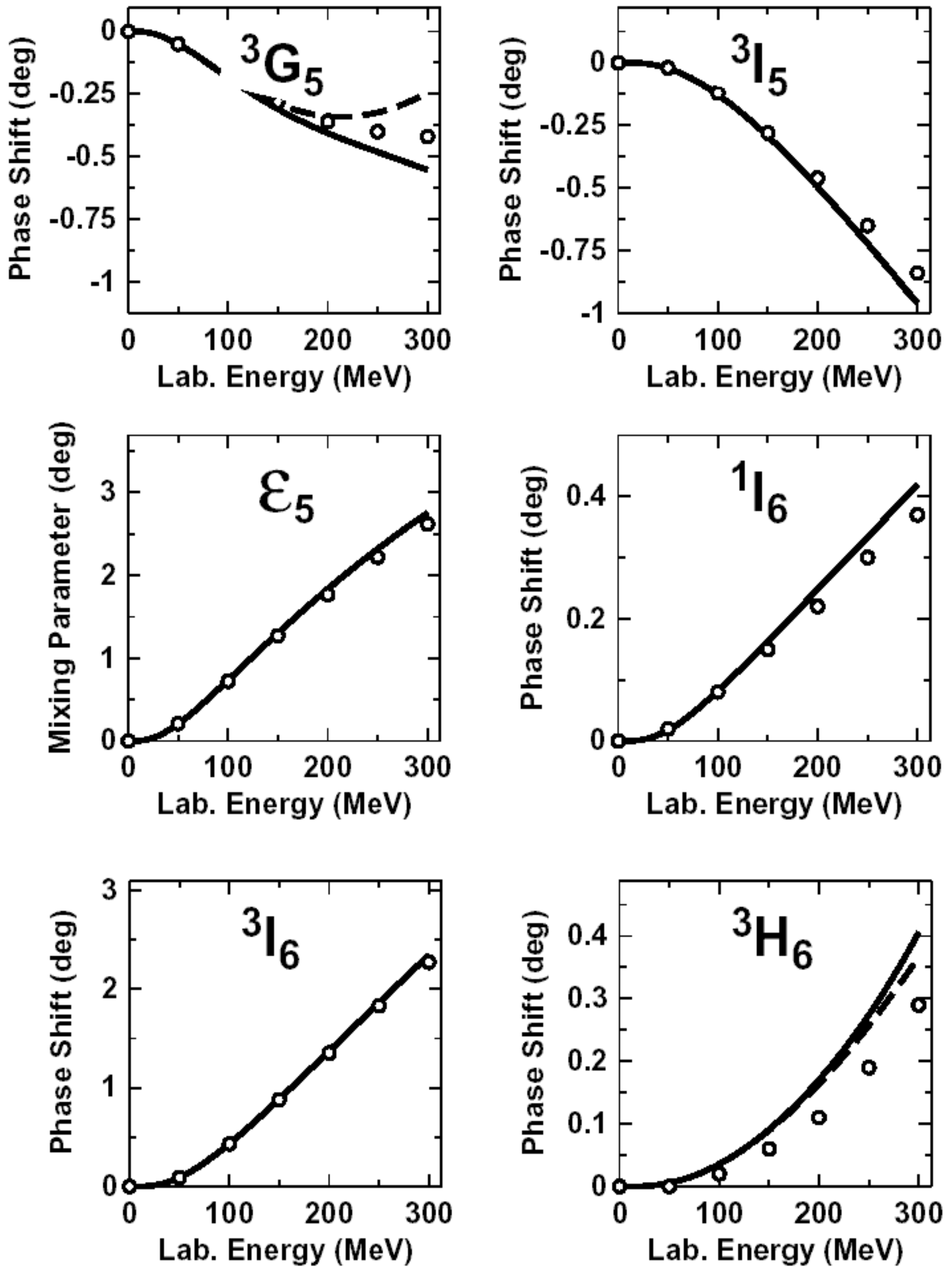


Mixing Parameter (deg)
Lab. Energy (MeV)
ε_1
Phase Shift (deg)
$^1D_2$
$^3D_2$
$^3P_2$
$^3F_2$
ε_2


1F3
3F3
3D3
3G3
ε3
1G4
Phase Shift (deg)
Mixing Parameter (deg)
Lab. Energy (MeV)



$^3G_4$
$^3F_4$
$^3H_4$
ε_4
$^1H_5$
$^3H_5$
Phase Shift (deg)
Mixing Parameter (deg)
Lab. Energy (MeV)


$^3G_5$
$^3I_5$
ε_5
$^1I_6$
$^3I_6$
$^3H_6$
Phase Shift (deg)
Mixing Parameter (deg)
Lab. Energy (MeV)
0
100
200
300
-0.25
-0.5
-0.75
-1
1
2
3
0.1
0.2
0.3
0.4

• Deuteron properties

Deuteron properties in comparison to other potential models and the empirical data. (Deuteron binding energy B_d; asymptotic S state A_S; asymptotic D/S state η; deuteron radius r_d; quadrupole moment Q; D-state probability P_D.)

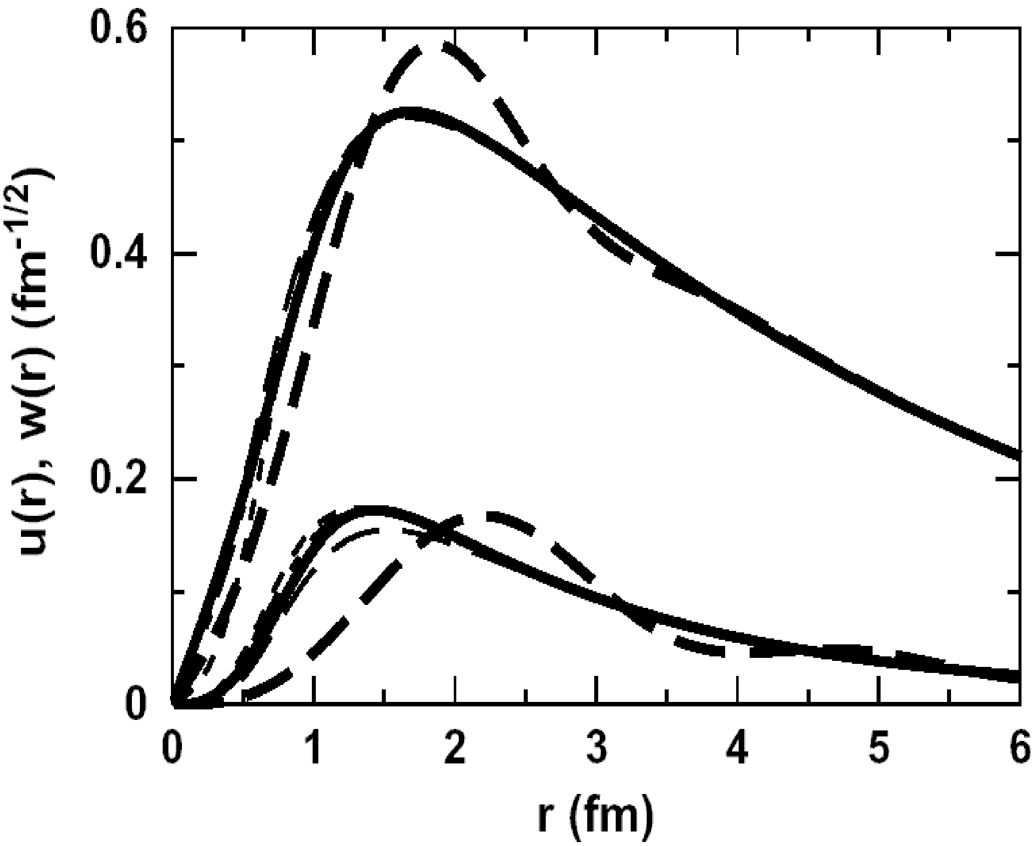
	This work	N^3LO	CD-Bonn	AV18	Empirical[a]
B_d (MeV)	2.224575	2.224575	2.224575	2.224575	2.224575(9)
A_S ($fm^{-1/2}$)	0.8844	0.8843	0.8846	0.8850	0.8846(9)
η	0.0254	0.0256	0.0256	0.0250	0.0256(4)
r_d (fm)	1.970[b]	1.978[b]	1.970[b]	1.971[b]	1.97535(85)
Q (fm^2)	0.283[c]	0.285[c]	0.280[c]	0.280[c]	0.2859(3)
P_D (%)	5.54	4.51	4.85	5.76	

[a]See Table XVIII of [R. Machleidt, Phys. Rev. **C63**, 024001 (2001)] for references.

[b]With meson-exchange currents (MEC) and relativistic corrections (RC)

[c]Including MEC and RC in the amount of 0.010 fm^2.

• Deuteron wave functions



Deuteron wave functions. The large waves are S-waves [$u(r)$] and the small waves are D-waves [$w(r)$]. The thick solid lines represent the waves generated from our present potential model while the thick dashed lines are from applying the chiral momentum space potential at N^3LO. The thin lines are based upon various conventional potential models.

7. Conclusions & Outlook

- Progress towards a quntitative chiral potential in configuration space for applications in microscopic nuclear few- and many-body calculations.

- Three-nucleon forces consistent with chiral effective field theory can be added.

- More consistent approach to microscopic nuclear structure.