Quantum Mechanics II

Homework Assignment 3

1. By using the variational method, find the first excited state for the harmonic oscillator. We know the Hamiltonian as $\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2 x^2$.

a. Use the trial function, $\Psi_1(x,\beta) = Cx\exp(-\beta x^2)$, where β is the variational constant, find the energy eigenvalue and the wave function.

b. The trial function should be $\Psi_1(x,\beta) = Cx\exp(-\beta x^2)$. The reason is that it has to be orthogonal to the eigenfunction for ground state. By doing the integration, make sure the orthogonality.

Note: The energy eigenvalue for the first excited state is $E_1 = \frac{3}{2}\hbar\omega$, and the eigenfunction is $\Psi_1 = \left(\frac{2}{\sqrt{\pi}}\right)^{\frac{1}{2}}\left(\frac{m\omega}{\hbar}\right)^{\frac{3}{4}} x\exp\left\{-(m\omega/2\hbar)x^2\right\}$. The result in a. should be the same as those.

2. Derive $-\frac{\hbar^2}{2m}\left(\frac{d^2\chi}{dr^2} - \frac{\lambda}{r^2}\chi\right) + V(r)\chi = \varepsilon\chi$ from

$-\frac{\hbar^2}{2m}\left(\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} - \frac{\lambda}{r^2}R\right) + V(r)R = \varepsilon R$ when you put $R(r) = \frac{1}{r}\chi(r)$.