Aharonov - Bohm effect

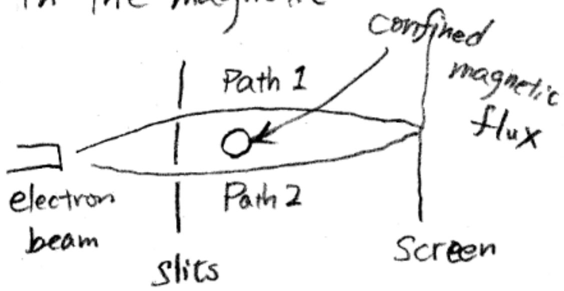
For free electrons,

$$i\hbar \frac{\partial}{\partial t} \psi_0 = -\frac{\hbar^2}{2m} \nabla^2 \psi_0$$

When the momentum becomes $\hat{p} \to \hat{p} - \frac{q}{c}\vec{A}$, the Schrödinger eq will be

$$i\hbar \frac{\partial}{\partial t} \psi = \frac{1}{2m}\left(-i\hbar\nabla - \frac{q}{c}\vec{A}\right)^2 \psi \quad -(1)$$

This is the equation for electrons moving in the magnetic fields.



Path 1 :

$$\psi_1 = \psi_1^0 \exp\left(\underbrace{i\frac{q}{\hbar c}\int_{C_1} \vec{A}\cdot d\vec{r}}_{\text{phase}}\right) \quad -(2)$$

Path 2 :

$$\psi_2 = \psi_2^0 \exp\left(\underbrace{i\frac{q}{\hbar c}\int_{C_2} \vec{A}\cdot d\vec{r}}_{\text{phase}}\right) \quad -(3)$$

The phase difference between path 1 and path 2 will be

$$\alpha_A = \frac{q}{\hbar c}\oint_C \vec{A}\cdot d\vec{r} \quad -(4)$$

The magnetic flux is expressed as

$$\Phi = \int_S \vec{B}\cdot d\vec{s} = \int_S \vec{\nabla}\times\vec{A}\cdot d\vec{s} = \oint_C \vec{A}\cdot d\vec{r} \quad \text{(Stokes theorem)} -(5)$$

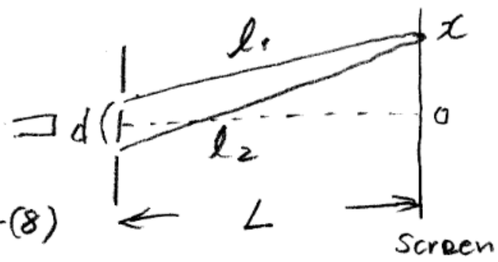
From eqs (4) and (5),

$$\alpha_A = \frac{q}{\hbar c}\Phi \quad -(6)$$

Besides existence of the vector potential, $\vec{A}$, there is a phase difference because of the path length difference. That is :

$$\alpha_0 = 2\pi\frac{S}{\lambda} \quad -(7)$$

The path length difference is



$$S = l_2 - l_1$$

$$= \sqrt{L^2 + (x + d/2)^2} - \sqrt{L^2 + (x - d/2)^2} \qquad \text{—(8)}$$

This can be approximated when $L \gg d, x$.

$$S \simeq \frac{xd}{L} \qquad \text{—(9)}$$

Use (7) and (9).

$$\alpha_0 = \frac{2\pi d}{L\lambda} x \qquad \text{—(10)}$$

The total phase difference is given as

$$\alpha_0 + \alpha_A = 0 \qquad \text{—(11)}$$

Set $\alpha_0 + \alpha_A = 0$ as the reference; then the shift will be

$$x_0 = -\frac{L\lambda}{2\pi d}\frac{q}{\hbar c}\Phi \qquad \text{where } \Phi = \oint_c \vec{A}\cdot d\vec{r} \qquad \text{—(12)}$$

[Use (6) and (10) and solve for x.]

No approximation:

Use (6), (7), and (11).

$$2\pi\frac{S(x)}{\lambda} + \frac{q}{\hbar c}\Phi = 0 \qquad \text{—(13)}$$

$$S(x) + \frac{\lambda q}{2\pi\hbar c}\Phi = 0 \qquad \text{—(14)}$$

Since $k = \frac{2\pi}{\lambda}$, we have

$$S(x) + \frac{q}{k\hbar c}\Phi = 0 \qquad \text{—(15)}$$

The transcendental eq. of x will be solved numerically.