Hirophysics.com

Probability of coin toss

For two-case probability (distribution), it follows Bernoulli distribution. Its natural conjugate distribution is Beta function. The expressions are:

$$B(x, y) = \frac{1}{y}\sum_{n=0}^{\infty}(-1)^n \frac{y^{\underline{n+1}}}{n!\,(x+n)} \qquad \left[x^{\underline{n}} = x(x-1)(x-2)\times \cdots (x-n+1)\right]$$

or $$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$$

The probability function of beta distribution $\Rightarrow$ $f(x, p, q) = \frac{1}{B(p,q)} x^{p-1}(1-x)^{q-1}$

With $B(p,q)$, the mean and variance of coin-toss are given by

$$\mu\,(\text{mean}) = \frac{p}{p+q}$$

$$\sigma^2\,(\text{variance}) = \frac{pq}{(p+q)^2(p+q+1)}$$

We assume that the prior distribution is $B(p,q)$.

After n trials, head occurs r times. This gives $\underline{B(p+r, q+n-r)}$ as the posterior probability:

Bayesian Update

$$\mu = \frac{p+r}{(p+r)+(q+n-r)} = \frac{p+r}{p+q+n}$$

$$\sigma^2 = \frac{(p+r)(q+n-r)}{\{(p+r)+(q+n-r)\}^2(p+q+n+1)} = \frac{(p+r)(q+n-r)}{(p+q+n)^2(p+q+n+1)}$$