Bayesian Statistics with Gaussian distribution

This is used for Statistical data which follow normal distribution.

- The prior probability:

$$\pi(\mu) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[- \frac{(\mu - \{\text{expected mean}\})^2}{2\sigma^2} \right]$$

- The likelihood:

$$f(D|\mu) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\sum_i - \frac{(x_i - \mu)^2}{2\sigma^2} \right]$$

- The posterior probability:

$$\pi(\mu|D) = f(D|\mu)\pi(\mu)$$

Derivation of the mean and variance with the likelihood

$$\pi(\mu) \propto \exp\left[\sum_i^n - \frac{(x_i-\mu)^2}{2\sigma^2} \right] \qquad \text{(The number of data is } n.\text{)}$$

$$= \exp\left[- \frac{(x_1-\mu)^2 + (x_2-\mu)^2 + (x_3-\mu)^2 + \cdots + (x_n-\mu)^2}{2\sigma^2} \right]$$

$$= \exp\left[- \frac{(x_1^2+x_2^2+x_3^2+\cdots+x_n^2) - 2\mu(x_1+x_2+x_3+\cdots+x_n) + n\mu^2}{2\sigma^2} \right]$$

$$= \exp\left[- \frac{\frac{1}{n}(x_1^2+x_2^2+x_3^2+\cdots x_n^2) - 2\mu\frac{1}{n}(x_1+x_2+x_3+\cdots x_n) + \mu^2}{\frac{1}{n}2\sigma^2} \right]$$

$$= \exp\left[- \frac{\overline{x^2} - 2\mu\bar{x} + \mu^2}{2\frac{\sigma^2}{n}} \right]$$ (where $\overline{x^2}$ and $\bar{x}$ are the average values.)

$$= \exp\left[- \frac{(\bar{x}-\mu)^2}{2\frac{\sigma^2}{n}} \right]$$