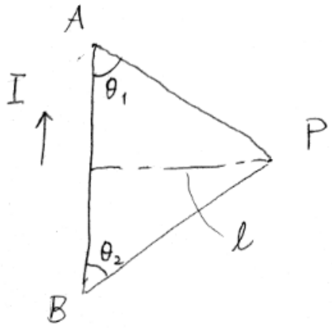
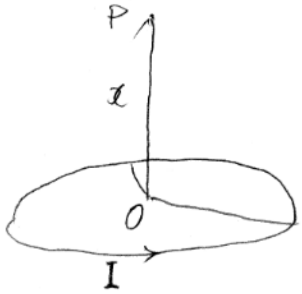
Hirophysics.com

Biot-Savart Law: Strategy to

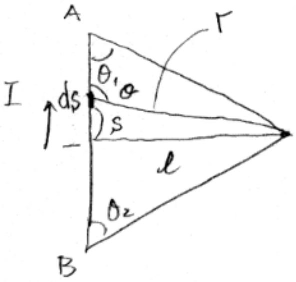
Find the magnetic field at point P.

Step ① Determine the perpendicular distance from the current source.

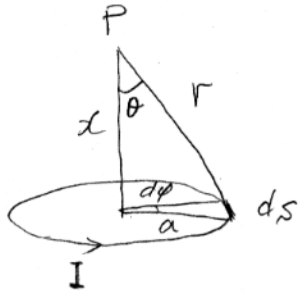




Step ② Find the arbitrary distance 'r' between source and 'P.'



$\frac{s}{r} = \cos(\pi - \theta)$



Step ③ Use 'ds' for the infinitesimal current, but express it in terms of infinitesimal angle, $d\theta$ or $d\varphi$.

$s = r\cos(\pi - \theta) = -r\cos\theta$ $\qquad$ $ds = a\,d\varphi$

Step ④ Express 'r' in terms of the other variables.

$r = \frac{l}{\sin(\pi - \theta)} = \frac{l}{\sin\theta}$ $\qquad$ $r = \sqrt{x^2 + a^2}$

Step ⑤ Calculate ds.

$s = \frac{l}{\sin\theta}(-\cos\theta) = -l\cot\theta$ $\qquad$ $ds = a\,d\varphi$

$ds = \frac{l}{\sin^2\theta}\,d\theta$

Step ⑥ Plug ds and r into Biot-Savart Law,

$$B = \frac{\mu_0 I}{4\pi}\int \frac{ds \sin\theta}{r^2}.$$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{\frac{l}{\sin^2\theta}\, d\theta \sin\theta}{\frac{l^2}{\sin^2\theta}}$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{\sin\theta\, d\theta}{l}$$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{a\, d\varphi \sin\theta}{x^2 + a^2}$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{a\, d\varphi}{x^2 + a^2}\frac{a}{r} \quad \left(\because \sin\theta = \frac{a}{r}\right)$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{a^2\, d\varphi}{(x^2 + a^2)^{3/2}}$$

Step ⑦ Integrate the eqs. in an appropreate condition.

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta_2}^{\pi-\theta_1} \frac{\sin\theta\, d\theta}{l}$$

$$= \frac{\mu_0 I}{4\pi l}\Big[-\cos\theta\Big]_{\theta_2}^{\pi-\theta_1}$$

$$= \frac{\mu_0 I}{4\pi l}\left\{-\cos(\pi-\theta_1) + \cos\theta_2\right\}$$

$$= \frac{\mu_0 I}{4\pi l}(\cos\theta_1 + \cos\theta_2)$$

$$B = \frac{\mu_0 I}{4\pi}\frac{a^2}{(x^2+a^2)^{3/2}}\int_0^{2\pi} d\varphi$$

$$= \frac{\mu_0 I}{4\pi}\frac{a^2}{(x^2+a^2)^{3/2}}\, 2\pi$$

$$= \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$$