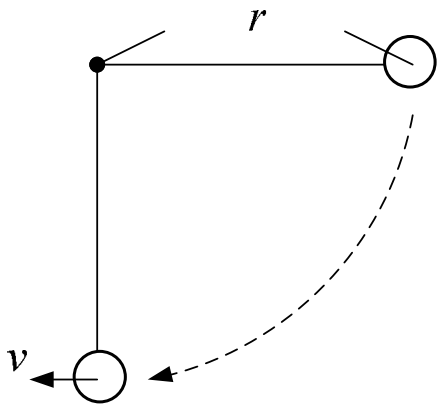
Finding the velocity of pendulum at the bottom

Problem: A simple pendulum whose length is *r* swings from the vertical height as shown. What is the velocity of the pendulum bob at the bottom?



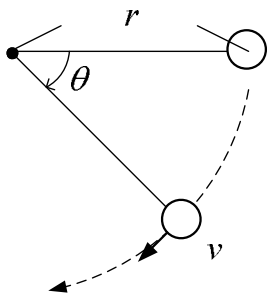
Solution 1: One can use the conservation of energy to find the velocity.

$$\frac{1}{2}mv^2 = mgr$$

Thus, $v = \sqrt{2gr}$

Solution 2: A kinematical equation, $v^2 = v_0^2 + 2a(x - x_0)$, can also be used. Since the initial velocity is zero and the displacement can be expressed by the radius and angle. Namely, $x - x_0 = r\theta$. The acceleration depends on the angle and written as $a = g\sin\theta$. Then, the original kinematical equation becomes $v^2 = 2(g\sin\theta)r\theta$.

Integrate it in terms of the angle.



$$v^2 = 2gr\int_0^{\frac{\pi}{2}} \theta\sin\theta\, d\theta$$

$$= 2gr\left[\sin\theta - \theta\cos\theta\right]_0^{\frac{\pi}{2}}$$

$$= 2gr\left[(1-0) - \left(\frac{\pi}{2}\cdot 0 - 0\cdot 1\right)\right]$$

$$= 2gr$$

Therefore, $v = \sqrt{2gr}$.