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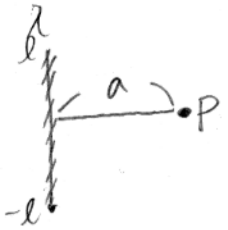
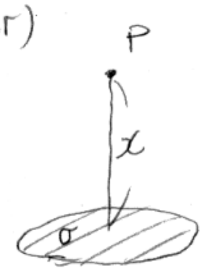
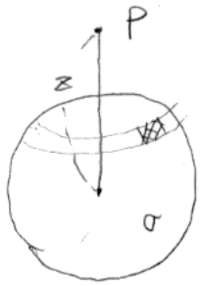
A strategy of how to obtain each potential and electric field.

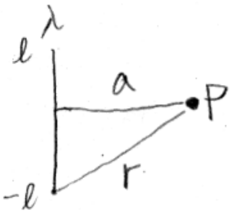
Step ① Find the distance between (perpendicular) a point and charged material.

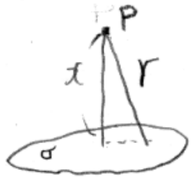




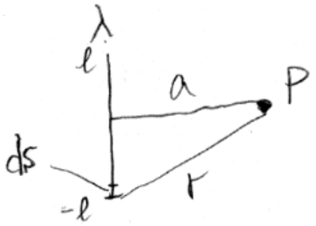


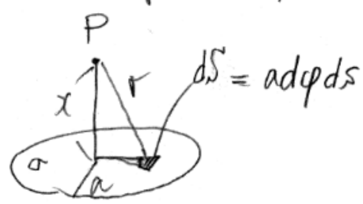
Step ② Assume the distance between 'P' and an arbitrary point on the charged material as 'r'.

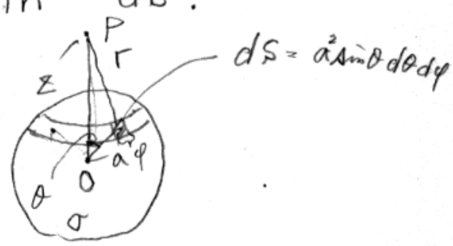




Step ③ Express the infinitesimal quantity in ds.







Step ④ Represend 'r' in terms of 'S'

$$r = \sqrt{S^2 + a^2} \qquad r = \sqrt{S^2 + x^2} \qquad r = \sqrt{a^2 + z^2 - 2az\cos\theta}$$

Step ⑤ Calculate the potential at P based on $\boxed{V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, ds}{r}}$

$$V = \frac{1}{4\pi\varepsilon_0}\int_{-l}^{l} \frac{\lambda\, dS}{\sqrt{S^2 + a^2}} \qquad V = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{a} \frac{\sigma S\, dS\, d\varphi}{\sqrt{S^2 + x^2}} \qquad V = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{\pi} \frac{\sigma a^2 \sin\theta\, d\theta\, d\varphi}{\sqrt{a^2 + z^2 - 2az\cos\theta}}$$

$\downarrow$

Put $t = -\cos\theta$

Step ⑥ Find the electric field.

Take derivative with respect to the perpendicular unchanged distance to point 'P.'

$$E = -\nabla V = -\frac{\partial}{\partial a} V \qquad E = -\frac{\partial}{\partial x} V \qquad E = -\frac{\partial}{\partial a} V$$