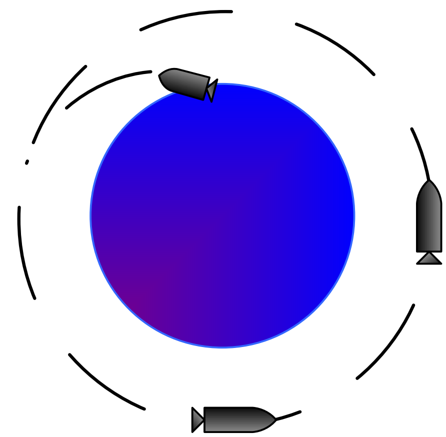
The first, second, and third cosmic velocities and black holes

The first cosmic velocity is known as the orbital velocity, which is the least velocity of a projectile to keep the orbit around a celestial body. When the centripetal and gravitational forces are in equilibrium, the object can hold the orbit. Let's consider the Earth's surface.



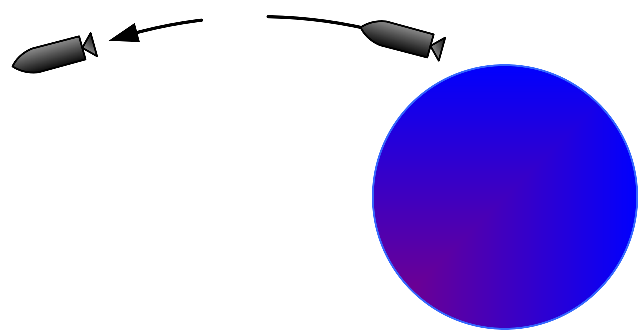
$$\frac{mv_1^2}{r} = G\frac{mM_E}{r^2}$$

Then, this velocity is obtained as

$$v_1 = \sqrt{\frac{GM_E}{r}} = 7.91\times10^3 \text{ m/s}$$

where G is the gravitational constant, 6.67×10^{-11} m^3 kg^{-1} S^{-2}. M_E is the mass of the Earth, 5.972×10^{24} kg; and r is the Earth's radius, 6371×10^3 m.

The second cosmic velocity is known as the escape velocity. This is the velocity that escapes from the gravitational field of a celestial body. This will be obtained by the energy conservation.

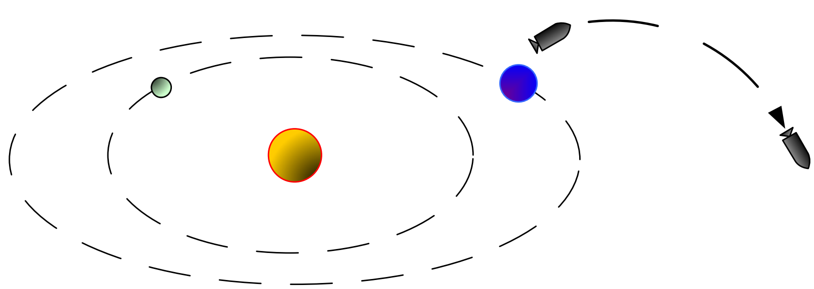


$$\frac{1}{2}mv_2^2 - \frac{GM_E m}{r} = 0$$

Thus, we can calculate it for Earth's surface.

$$v_2 = \sqrt{\frac{2GM_E}{r}} = 11.2\times10^3 \text{ m/s}$$

The third cosmic velocity is the velocity that can escape from the gravitational field of the solar system. Consider the orbit of the Earth around the Sun and this radius is $r_E = 1.50\times10^{11}$ m. The velocity is calculated from the energy conservation with the gravity of the Sun.



The mass of the Sun is $M_S = 1.99\times10^{30}$ kg. The velocity will become

$$v_S = \sqrt{\frac{2GM_S}{r_E}} = 42.1\times 10^3 \ \text{ m/s}$$

This is the velocity from the reference frame of the Sun; therefore, we have to subtract the orbital velocity from the above velocity. Earth's orbital velocity can be obtained from the centripetal force and gravitational force from the Sun.

$$v_{\text{orbit}} = \sqrt{\frac{GM_S}{r_E}} = 29.8\times 10^3 \ \text{ m/s}$$

Thus, the escape velocity from the orbit is

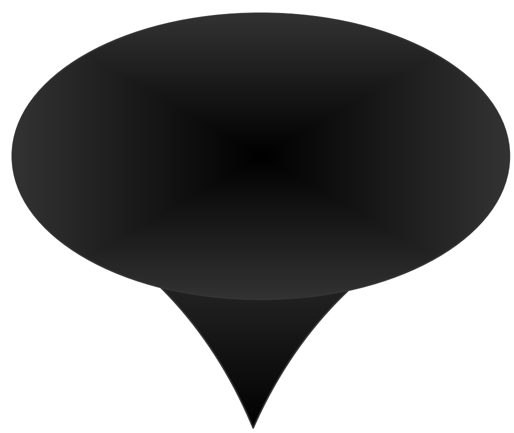
$$v_{\text{o-S}} = v_S - v_{\text{orbit}} = 12.3\times 10^3 \ \text{ m/s}$$

Now, consider the velocity from the surface of the Earth, which becomes the third cosmic velocity.

$$\frac{1}{2}mv_3^2 - \frac{GM_E m}{r} = \frac{1}{2}mv_{\text{o-S}}^2$$

$$v_3 = \sqrt{\frac{2GM_E}{r} + v_{\text{o-S}}^2} = 16.7\times 10^3 \text{ m/s}$$

Black holes and Schwarzschild radius



We can consider the second cosmic velocity.

$$v = \sqrt{\frac{2GM}{r}}$$

This is the velocity to escape the gravitational potential energy due to a celestial body the mass of *M*. Let's think of such planet or star which even the speed of light cannot escape from. Replace v in the above with the speed of light, *c*. Then, solve for the radius of the star.

$$c = \sqrt{\frac{2GM}{r}}$$

$$\Rightarrow r = \frac{2GM}{c^2}$$

This is known as Schwarzschild radius that is the threshold of becoming a black hole that sucks everything. A black hole is made from contraction by explosion of a large star. The contraction is induced by its gravitational force. The star still keeps the same mass; therefore, it gets denser to obtain more gravitational force. For example, what is the radius when the Sun becomes a black hole? In other words, what is the Schwarzschild radius for the Sun? The mass is 1.99×10^{30} kg. The speed of light is 3.00×10^{8} m/s. Thus, The Schwarzschild radius will be calculated as

$$r = \frac{2 \cdot 6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{\left(3.00 \times 10^{8}\right)^{2}} = 2949 \quad \mathrm{m}$$

Since the radius of the sun is 6.96×10^{8} m. In order to be a black hole, it must be shrunk into one millionth of the original size.