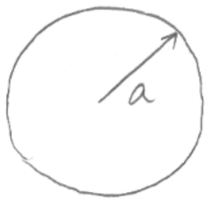
# Gauss' Law

$$\int_S \vec{E}\cdot d\vec{s} = \frac{1}{\epsilon_0}\int_V \rho\, dV$$

For the sphere,




$$\rho = \begin{cases} \frac{3Q}{4\pi a^3} & r \geq a \\ \frac{3Q}{4\pi a^3}\left(\frac{r}{a}\right)^3 & r < a \end{cases}$$

Therefore,

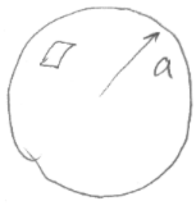
$$4\pi r^2 E = \frac{1}{\epsilon_0}\frac{3Q}{4\pi a^3}\int_0^a\int_0^\pi\int_0^{2\pi} r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= \frac{Q}{\epsilon_0}$$

$$\therefore \quad E = \frac{Q}{4\pi\epsilon_0 r^2} \qquad (r \geq a)$$

$$= \frac{Qr}{4\pi\epsilon_0 a^3} \qquad (r < a)$$

For the thin-walled sphere,




$$\rho = \begin{cases} \frac{Q}{4\pi a^2} & (r \geq a) \\ 0 & (r < a) \end{cases}$$

$$\int_0^a E\, ds = \frac{Q}{4\pi a^2 \epsilon_0}\int_0^\pi\int_0^{2\pi} a^2 \sin\theta\, d\theta\, d\phi$$

$$4\pi r^2 E = \frac{Q}{\epsilon_0}$$

$$\therefore \quad E = \frac{Q}{4\pi r^2 \epsilon_0} \qquad (r \geq a)$$

$$= 0 \qquad (r < a)$$