Classical Mechanics

1. Oscillatory Systems

a. Hamiltonian =

- harmanic oscillation

$$T = \frac{1}{2} m \dot{x}^2 \quad ; \quad U = \frac{1}{2} k x^2 = \frac{1}{2} m \omega^2 x^2$$

- simple pendulum

$$T = \frac{1}{2} m (l\dot{\theta})^2 \; ; \quad U = mgl(1-\cos\theta)$$

- spherical pendulum

$$T = \frac{1}{2} m \cdot \left\{ (l\dot{\theta})^2 + (l\dot{\varphi}\sin\theta)^2 \right\} \; ; \quad U = mgl(1-\cos\theta)$$

$*$ $x = l \sin\theta \cos\varphi \; ; \; y = l \sin\theta \sin\varphi \; ; \; z = l\cos\theta$

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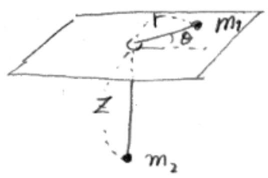
example.

procedure 1. Find the kinetic energies for m_1 and m_2.



$$T_1 = \frac{1}{2} m_1 (\dot{x}^2 + \dot{y}^2)$$

$$= \frac{1}{2} m_1 \left\{ (r\cos\theta)'^2 + (r\sin\theta)'^2 \right\}$$

$$= \frac{1}{2} m_1 (\dot{r}^2 + r^2\dot{\theta}^2)$$

$$T_2 = \frac{1}{2} m_2 \dot{z}^2 = \frac{1}{2} m_2 (l-r)'^2 = \frac{1}{2} m_2 (-\dot{r})^2 = \frac{1}{2} m_2 \dot{r}^2$$

procedure 2. Find the potential.

$$U = m_2 g z$$

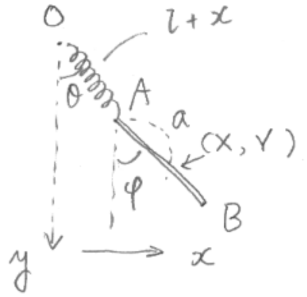
procedure 3. Make the Lagrangian and the eq. of motion.

$$L = T_1 + T_2 - U = \frac{1}{2} m_1 (\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{2} m_2 \dot{r}^2 - m_2 g (r - l)$$

But $r + z = l$. Make use of $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$

$$\therefore \; (m_1 + m_2)\ddot{r} - m_1 r \dot{\theta}^2 + m_2 g = 0 \; ; \quad \frac{d}{dt}(m_1 r^2 \dot{\theta}) = 0$$

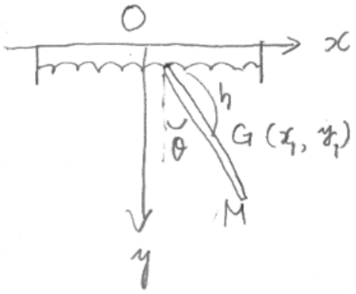
For double pendulum,



$$X = (l+x)\sin\theta + a\sin\varphi$$
$$Y = (l+x)\cos\theta + a\cos\varphi$$

$$T = \frac{1}{2}M(\dot{X}^2+\dot{Y}^2) + \frac{1}{2}\left(\frac{Ma^2}{3}\right)\dot{\varphi}^2$$

$$U = -MgY + \frac{1}{2}kx^2$$

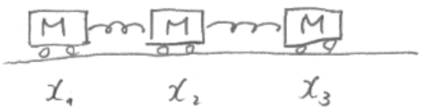


$$\begin{cases} x_1 = x + h\sin\theta \\ y_1 = h\cos\theta \end{cases}$$

$$T = \frac{1}{2}\left(M(\dot{x}_1^2+\dot{y}_1^2) + I\dot{\theta}^2\right)$$

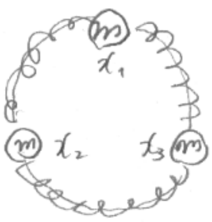
$$U = -Mgy_1 + \frac{1}{2}kx^2$$

If you use a pendulum instead of the bar, the term of moment of inertia is zero.



$$T = \frac{1}{2}M(\dot{x}_1^2+\dot{x}_2^2+\dot{x}_3^2)$$

$$U = \frac{1}{2}k\left[(x_2-x_1)^2+(x_3-x_2)^2\right]$$



$$T = \frac{1}{2}m(\dot{x}_1^2+\dot{x}_2^2+\dot{x}_3^2)$$

$$U = \frac{1}{2}k\left[(x_1-x_2)^2+(x_2-x_3)^2+(x_3-x_1)^2\right]$$

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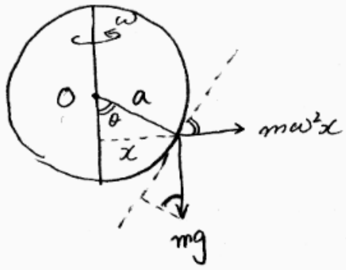

equilibrium point ?

At equilibrium point, the tangent components of centrifugal force and gravitational force.

$$mg\sin\theta = m\omega^2 x\cos\theta$$

but $x = a\sin\theta$, so

$$\sin\theta\,(g - a\omega^2\cos\theta) = 0$$

$\therefore$ $g < a\omega^2$, $\theta = 0$ and $\theta = \cos^{-1}(g/a\omega^2)$

$g \geqq a\omega^2$, $\theta = 0$.