Perturbation theory

System = 1-dimensional S.H.O.

Perturbation potential = $V = \frac{1}{2}\epsilon m\omega^2 x^2$ $(\epsilon \ll 1)$

Find the matrix elements for V_{00} and V_{20}.

Using $x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$,

we have $x^2 = \frac{\hbar}{2m\omega}(aa + aa^\dagger + a^\dagger a + a^\dagger a^\dagger)$.

Then $\langle 0|x^2|0\rangle = \frac{\hbar}{2m\omega}\{\langle 0|aa^\dagger|0\rangle + \langle 0|a^\dagger a|0\rangle\}$

($\langle 0|aa^\dagger|0\rangle \hookrightarrow 1$, $\langle 0|a^\dagger a|0\rangle \hookrightarrow 0$)

$$= \frac{\hbar}{2m\omega}$$

$$\langle 2|x^2|0\rangle = \frac{\hbar}{2m\omega}\{\langle 2|a^\dagger a^\dagger|0\rangle\}$$

$$= \frac{\hbar}{2m\omega}\sqrt{2}$$

As a result, we obtain

$$V_{00} = \frac{1}{2}\epsilon m\omega^2\langle 0|x^2|0\rangle$$

$$= \frac{\epsilon\hbar\omega}{4}.$$

$$V_{20} = \frac{1}{2}\epsilon m\omega^2\langle 2|x^2|0\rangle$$

$$= \frac{\sqrt{2}\epsilon\hbar\omega}{4}$$