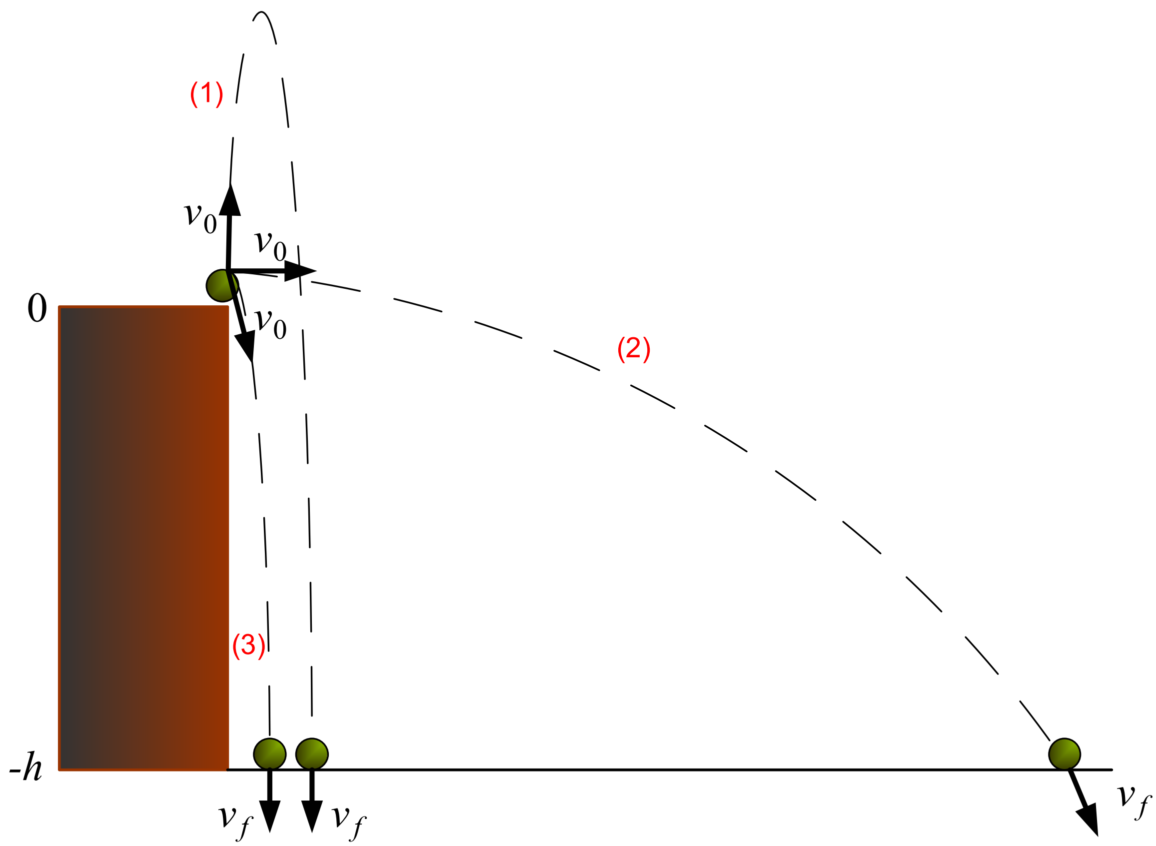
Which projectile motion makes the fastest final velocity?



There are three orbits of projectiles. Projectile (1) is thrown vertically upward. Projectile (2) is thrown horizontally. Projectile (3) is thrown vertically downward. The initial speed of three cases is the same. The height of the building is *h*.

Let's calculate each final velocity with kinematic equations.
Projectile (1):

Use $v_f^2 - v_0^2 = -2gh$. Therefore,

$$v_f = \sqrt{v_0^2 - 2g(-h)} = \sqrt{v_0^2 + 2gh}$$

Projectile (2):
For the *x* direction, the velocity is constant, so the final velocity is equal to the initial velocity.

$$v_{xf} = v_0$$

For the y direction, there is no initial velocity.

$$v_{yf}^2 = -2g(-h)$$

$$\Rightarrow \; v_{yf} = \sqrt{2gh}$$

Thus, the resultant velocity is given as

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{v_0^2 + 2gh}$$

Projectile (3):

The initial velocity is directed negatively.

$$v_f = \sqrt{(-v_0)^2 - 2g(-h)} = \sqrt{v_0^2 + 2gh}$$

We can conclude that all of the cases have the same final velocity!

Needless to say, we can obtain the same result if using conservation of energy. Namely,

$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_f^2$$

For all the cases, we can use only v_0 because the energy equation is a scalar equation which does not consider the directions.

However, each time (t_1, t_2, and t_3) to reach the ground is <u>different</u>. The time for the projectile motion is determined by the vertical motion. Thus, we use

$$y = y_0 + v_0 t - \frac{1}{2}gt^2 \; \Rightarrow \; -\frac{1}{2}gt^2 + v_0 t + h = 0$$

Projectile (1):

$$v_0 > 0$$

Projectile (2):

$$v_0 = 0$$

Projectile (3):

$$v_0 < 0$$

Apparently, if the other conditions are equal, we would obtain $t_1 > t_2 > t_3$.