○ Time-dependent perturbation theory.

Time-dependent perturbation term

$$H'(t) = exF \sin\omega t \qquad (H'(t)=0 \text{ when } t<0)$$

Find $a_k^{(1)}(t)$ and the transition probability.

The perturbation hamiltonian can be rewritten as

$$H'(t) = eFx\,\frac{e^{i\omega t}-e^{-i\omega t}}{2i}.$$

Using

$$a_k^{(1)}(t) = \frac{1}{i\hbar}\int_0^t H'_{k0}(t')\,e^{i\omega_{k0}t'}\,dt',$$

we obtain

$$a_k^{(1)}(t) = \frac{eF}{i\hbar}x_{k0}\int_0^t \frac{e^{i\omega t'}-e^{-i\omega t'}}{2i}\,e^{i\omega_{k0}t'}\,dt'$$

$$= \frac{ieF}{2\hbar}x_{k0}\left[\frac{e^{i(\omega_{k0}+\omega)t}-1}{\omega_{k0}+\omega} - \frac{e^{i(\omega_{k0}-\omega)t}-1}{\omega_{k0}-\omega}\right]$$

where $x_{k0} = \int u_k^* x\, u_0\, d\vec{r}$.

u_k and u_0 are the unperturbed eigenfunctions.

Therefore,

$$|a_k^{(1)}(t)|^2 = \frac{e^2F^2|x_{k0}|^2}{4\hbar^2}\left[\frac{4\sin^2\{(\omega_{k0}+\omega)/2\}t}{(\omega_{k0}+\omega)^2} + \frac{4\sin^2\{(\omega_{k0}-\omega)/2\}t}{(\omega_{k0}-\omega)^2}\right.$$

$$\left. + \frac{8}{(\omega_{k0}+\omega)(\omega_{k0}-\omega)}\sin\frac{1}{2}(\omega_{k0}+\omega)t\cdot\sin\frac{1}{2}(\omega_{k0}-\omega)t\,\cos\omega t\right]$$