Question:

What is the time to stick together two objects due to only gravitational force?

Solution:

The gravitational force is an attractive force given as

$$F = -G\frac{Mm}{r^2}$$

Assume that the small object moves toward the large one. The equation of motion is

$$F = m\frac{d^2r}{dt^2}$$

Thus,

$$\sum F = -G\frac{Mm}{r^2} = m\frac{d^2r}{dt^2}$$

Multiply both sides by dr/dt, and integrate in terms of t.

$$\int \frac{dr}{dt}\frac{d^2r}{dt^2}dt = -GM\int \frac{1}{r^2}\frac{dr}{dt}dt$$

For the right hand side, dt can be cancelled out. Then,

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = -GM\int \frac{1}{r^2}dr + C$$

$$\Rightarrow \frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{GM}{r} + C$$

The left hand side is a little tricky, but if you take derivative of $(1/2)(dr/dt)^2$ with respect to t, you will obtain $(dr/dt)(d^2r/dt^2)$. If R is infinite, the gravitational force and velocity will be zero. Thus, the constant, C, can also be zero. The above equation will be modified as follows:

$$\frac{dr}{dt} = \sqrt{\frac{2GM}{r}}$$

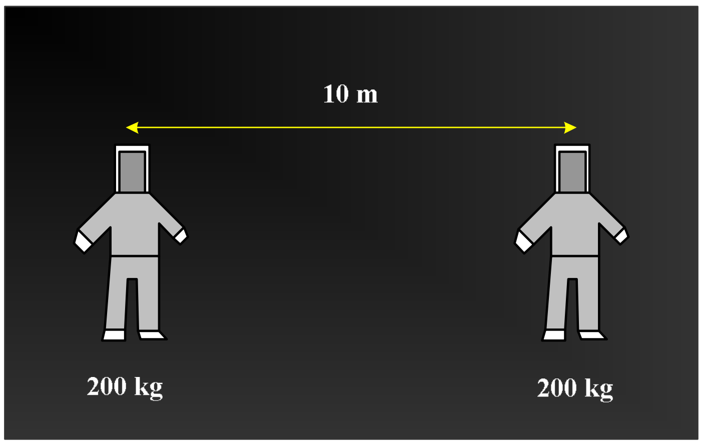
$$\Rightarrow \frac{dr}{dt}\sqrt{\frac{r}{2GM}} = 1$$

Integrate both sides in terms of t.

$$\Rightarrow t = \frac{2}{3}\sqrt{\frac{r^3}{2GM}} + C'$$

When the two objects stick together ($r = 0$) as $t = 0$, the constant, C', is zero. This equation can also describe how long it takes to collapse into a point for a celestial body.

Suppose there are two astronauts in space. Their masses are equally 200 kg. (They are wearing spacesuits.) They are separated by 10 m. If there is only gravitational force between them, how long will it take to get together?



Simply plug numbers in the above formula.

$$t = \frac{2}{3}\sqrt{\frac{(10\text{ m})^3}{2\times 6.67\times 10^{-11}\,\text{m}^3/(\text{kg}\,\text{s}^2)\times 200\,\text{kg}}} = 129{,}067\,\text{s}$$

This is about 36 hours.