Centrifugal Barrier

Centrifugal barrier appears in a spherically symmetric system on mechanics.[1] Especially for nuclear physics, this plays an important role to deal with the scattering problem. Let us take a two nucleon scattering system that is spherically symmetric. The radial part of Schrödinger equation becomes:

$$-\frac{\hbar^2}{2\mu}\left\{\frac{1}{r^2}\frac{d}{dr}r^2\frac{r}{dr}-\frac{l(l+1)}{r^2}\right\}R_l(k,r)+V(r)R_l(k,r)=ER_l(k,r)$$

The $V(r)$ is a central potential, such as Coulomb-like potential energy, but for nucleon scattering, it is highly complicated because the forces are divided into three parts and the geometrical information has to be involved into the potential. The μ represents the reduced mass. The term, $-\frac{l(l+1)}{r^2}$, is called centrifugal barrier since it effectively behaves as repulsive force (due to the negative sign). Thus, we can extract the effective potential from the above equation:

$$\tilde{V}(r)=V(r)+\frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}$$

When the orbital angular momentum, l, becomes large, the centrifugal barrier causes a large effect in scattering. If the scattering energy is less than 10 mega electron volts, $E<10\text{MeV}$, nucleon-nucleon scattering is essentially given by S-wave; namely, $l=1$. In other words, the centrifugal barrier does not make a difference due to the small energy.

General Descriptions of Potential Barrier

When the kinetic energy of a particle is less than the potential energy of the system, the motion will not exceed the potential wall. This is a general concept of the potential barrier. Let us think of a system with a central force in classical mechanics. The hamiltonian is expressed as follows:

$$H=\frac{p_r}{2m}+\frac{L^2}{2mr^2}-\frac{\alpha}{r}$$

where p_r, L, and $-\frac{\alpha}{r}$ are the linear momentum, angular momentum, and central potential, respectively. The first term of the hamiltonian, $\frac{p_r}{2m}$ is the linear kinetic energy of the system. The second term is the rotational kinetic energy; however, the variable, r, plays a tricky role. As it approaches zero, the term becomes large as if it becomes a core repulsive potential. Therefore, the effective potential energy can be recognized as

$$\tilde{U}(r) = \frac{L^2}{2mr^2} - \frac{\alpha}{r}$$

We assume that the angular momentum is constant. Each term can be plotted as shown: Obviously, the force can be derived from the term called centrifugal potential, $\frac{L^2}{2mr^2}$. Let

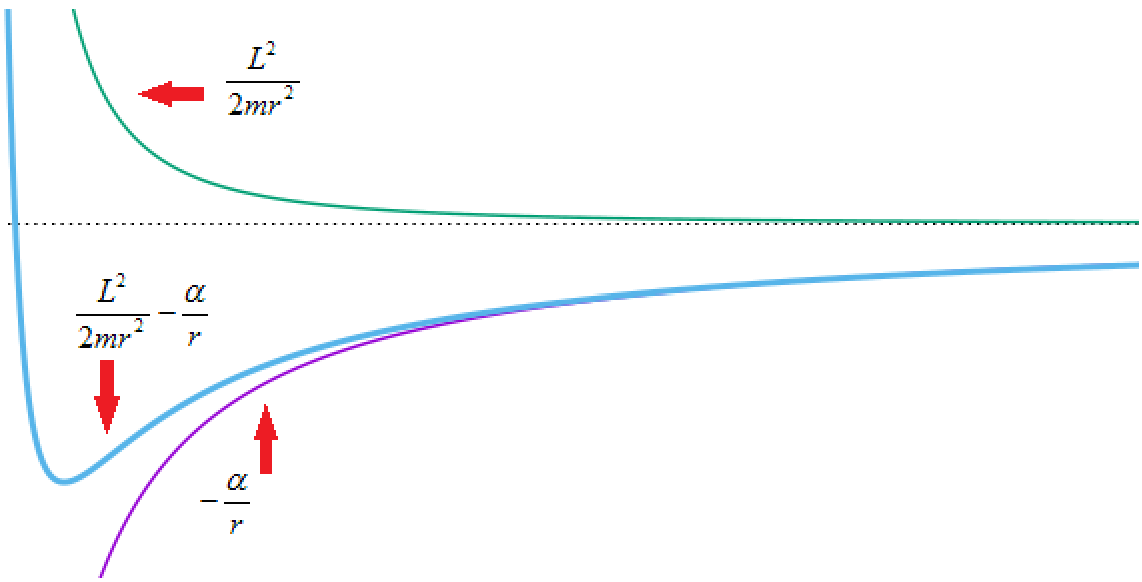

Figure 1: Each potential term and its composite

us take derivative of the potential with respect to r.

$$\frac{\partial}{\partial r} U(r)_{\text{cfb}} = \frac{\partial}{\partial r} \frac{L^2}{2mr^2} = -2\frac{L^2}{2mr^3}$$

The angular momentum, L, is assumed to be a constant and can also be expressed as mvr.

$$\frac{\partial}{\partial r} U(r)_{\text{cfb}} = -\frac{(mvr)^2}{mr^3} = -\frac{mv^2}{r}$$

This is the classical centrifugal force which is the fictitious one that occurs in rotational reference frames.

References

[1] J. J. Sakurai, *Advanced Quantum Mechanics*. Addison-Wesley,1967

Presented by Hirophysics.com