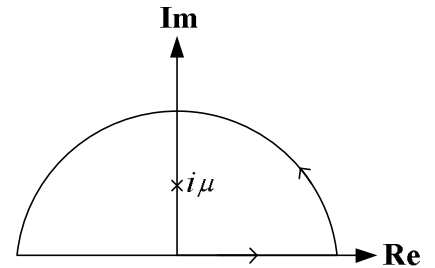


Infinite integral with a half circle contour

Integrate the following function.

$$\begin{aligned} I &= \int_0^{\infty} \frac{k^2 \exp[ikr]}{k^2 + \mu^2} dk \\ &= \frac{1}{2} \int_{-\infty}^{\infty} \frac{k^2 \exp[ikr]}{k^2 + \mu^2} dk \end{aligned}$$



From the denominator, the poles are $\pm i\mu$. There is only $i\mu$ as the pole in the contour. We have the residue:

$$\text{Res}[f, i\mu] = \lim_{z \rightarrow i\mu} \frac{z^2 \exp[izr]}{z + i\mu} = \frac{-\mu^2 \exp[-\mu r]}{2i\mu}$$

The value of integral is from the residue theorem as follows:

$$I = \frac{1}{2} 2\pi i \frac{-\mu^2 \exp[-\mu r]}{2i\mu} = -\frac{\mu\pi \exp[-\mu r]}{2}$$