Important properties of deuteron

We can categorize each state corresponding to the related conditions: The bound state,

States	$^1S_0$	$^1P_1$	$^1D_2$	$^3S_1 +^3 D_1$
Parity	$+$	$-$	$+$	$+$
Particle exchange	$-$	$+$	$-$	$+$
Interactions	nn, np, and pp	np	nn, np, and pp	np (deuteron)

$^3S_1 +^3 D_1$, illustrates the deuteron. The deuteron has no excited state partly because of its small binding energy. S, P, D, correspond to the orbital angular momentum, $L = 0, 1, 2$. The number of left superscript expresses the spin multiplet, $2S + 1$ and the right subscript is the total angular momentum, J.

The reason for mixed states of deuteron wave functions

The magnetic moment of proton is $\mu_p = +2.7927$ and that of neutron is $\mu_n = -1.91315$. The experimental deuteron magnetic dipole moment is $\mu_d = +0.857393$, which is not equal to $\mu_p + \mu_n$. The ground state of the deuteron has $J = 1$. Taking the correspondent states, $^1P_1$, $^3P_1$, $^3S_1$, and $^3D_1$. For the same parity, we can have two pairs, $^1P_1$ and $^3P_1$; $^3S_1$ and $^3D_1$. Only $^3S_1$ and $^3D_1$ gives the proper result with $^3S_1$-state = 96% and $^3D_1$-state = 4%. Due to this mixture, the potential energy additionally requires spatial description with a spherical tensor of rank 2:

$$V_T = \frac{3}{r^2}(\sigma_1 \cdot r)(\sigma_2 \cdot r) - \sigma_1\sigma_2$$

The reason why the isospin must be zero for the deuteron

According to the deuteron potential, if the total isospin is $T = \pm 1$, only repulsive forces arise. Therefore only pn case ($T = 0$) makes the bound state (deuteron).

The other properties related to the wave functions

Each state corresponds to each wave function as follows: The asymptotic behaviors of the

State		Wave Function
$^3S_1$	$\longrightarrow$	$u(r)$
$^3D_1$	$\longrightarrow$	$w(r)$

wave functions are known as

$$\begin{aligned} u(r) &\sim A_S \exp[-\gamma r] \\ w(r) &\sim A_D \exp[-\gamma r]\left[1 + \frac{3}{\gamma r} + \frac{3}{(\gamma r)^2}\right] \end{aligned}$$

The D/S state ratio is given as

$$\eta \equiv \frac{A_D}{A_D}$$

This ratio measured accurately in the asymptotic region of the wave function, and is a good test of the existence and correctness of the description of the long-range NN force in terms of the pion exchange theory.

The (electric) quadrupole moment and radius of the deuteron are described as

$$\begin{aligned} Q_d &= \frac{1}{20}\int_0^\infty drr^2 w(r)[\sqrt{8}u(r) - w(r)] \qquad \text{Quadrupole Moment} \\ r_d &= \frac{1}{2}\left\{\int_0^\infty drr^2[u^2(r) + w^2(r)]\right\}^{\frac{1}{2}} \qquad \text{Deuteron Radius} \end{aligned}$$

References

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