Important properties of deuteron

We can categorize each state corresponding to the related conditions: The bound state,

<table>
<thead>
<tr>
<th>States</th>
<th>$^1S_0$</th>
<th>$^1P_1$</th>
<th>$^1D_2$</th>
<th>$^3S_1 + ^3D_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parity</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>Particle exchange</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>Interactions</td>
<td>$nn$, $np$, and $pp$</td>
<td>$np$ $nn$, $np$, and $pp$</td>
<td>$np$ (deuteron)</td>
<td></td>
</tr>
</tbody>
</table>

$^3S_1 + ^3D_1$, illustrates the deuteron. The deuteron has no excited state partly because of its small binding energy. $S$, $P$, $D$, correspond to the orbital angular momentum, $L = 0, 1, 2$. The number of left superscript expresses the spin multiplet, $2S + 1$ and the right subscript is the total angular momentum, $J$.

The reason for mixed states of deuteron wave functions

The magnetic moment of proton is $\mu_p = +2.7927$ and that of neutron is $\mu_n = -1.91315$. The experimental deuteron magnetic dipole moment is $\mu_d = +0.857393$, which is not equal to $\mu_p + \mu_n$. The ground state of the deuteron has $J = 1$. Taking the correspondent states, $^1P_1$, $^3P_1$, $^3S_1$, and $^3D_1$. For the same parity, we can have two pairs, $^1P_1$ and $^3P_1$; $^3S_1$ and $^3D_1$. Only $^3S_1$ and $^3D_1$ gives the proper result with $^3S_1$-state = 96% and $^3D_1$-state = 4%. Due to this mixture, the potential energy additionally requires spatial description with a spherical tensor of rank 2:

$$V_T = \frac{3}{r^2}(\sigma_1 \cdot r)(\sigma_2 \cdot r) - \sigma_1 \sigma_2$$

The reason why the isospin must be zero for the deuteron

According to the deuteron potential, if the total isospin is $T = \pm 1$, only repulsive forces arise. Therefore only $pn$ case ($T = 0$) makes the bound state (deuteron).
The other properties related to the wave functions

Each state corresponds to each wave function as follows: The asymptotic behaviors of the wave functions are known as

\[ u(r) \sim A_S \exp[-\gamma \gamma r] \]
\[ w(r) \sim A_D \exp[-\gamma r] \left[ 1 + \frac{3}{\gamma r} + \frac{3}{(\gamma r)^2} \right] \]

The D/S state ratio is given as

\[ \eta \equiv \frac{A_D}{A_D} \]

This ratio measured accurately in the asymptotic region of the wave function, and is a good test of the existence and correctness of the description of the long-range NN force in terms of the pion exchange theory.

The (electric) quadrupole moment and radius of the deuteron are described as

\[ Q_d = \frac{1}{20} \int_0^{\infty} drr^2 w(r) [\sqrt{8} u(r) - w(r)] \quad \text{Quadrupole Moment} \]
\[ r_d = \frac{1}{2} \left\{ \int_0^{\infty} drr^2 [u^2(r) + w^2(r)] \right\}^{\frac{1}{2}} \quad \text{Deuteron Radius} \]

References

