How to obtain the magnetic moment

Write down the spin/flavor wave function for s proton with spin up.

A proton has three quarks, which are up, up and down. The wave function can explicitly be expressed as follows:

\[ |B \uparrow \rangle = \left| p : \frac{1}{2}, m_s = \frac{1}{2} \right\rangle = \frac{1}{2}(\uparrow\downarrow - \downarrow\uparrow)(udu - duu) + \frac{1}{2}(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)(uud - duu) \]
\[ + \frac{1}{2}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)(uud - duu) \frac{\sqrt{2}}{3} \]
\[ = \frac{2}{3\sqrt{2}} [u(\uparrow)u(\uparrow)d(\downarrow)] - \frac{1}{3\sqrt{2}} [u(\uparrow)u(\downarrow)d(\uparrow)] \]
\[ - \frac{1}{3\sqrt{2}} [u(\downarrow)u(\uparrow)d(\uparrow)] + \text{permutations (6 more terms)} \]

Now calculate the magnetic moment of the proton. The magnetic moment is described with spin operator.

\[ \mu = \frac{q}{mc} \vec{S} = \frac{q\hbar}{2mc} \]

The magnetic moments of up and down quarks are determined by the above relationship.

That is:

\[ \mu_{up} = \frac{3}{2} \frac{e\hbar}{2m_{up}c}, \quad \mu_{down} = -\frac{1}{3} \frac{e\hbar}{2m_{down}c} \]

The magnetic moment is given by

\[ \mu_B = \frac{2}{\hbar} \sum_{i=1}^{3} \langle B \uparrow | (\mu_i S_{iz}) | B \uparrow \rangle \]

Using the above result, we obtain the magnetic moment for a proton:

\[ \mu_p = \frac{3}{2} \left[ \frac{2}{9}(2\mu_{up} - \mu_{down}) + \frac{1}{18} \mu_{up} + \frac{1}{18} \mu_{down} \right] \]
\[ = \frac{1}{3}(4\mu_{up} - \mu_{down}) \]

The magnetic moment of a neutron is given as

\[ \mu_n = \frac{1}{3}(\mu_{up} - 4\mu_{down}) \]
References