How to obtain the magnetic moment[1]

Write down the spin/flavor wave function for s proton with spin up.

A proton has three quarks, which are up, up and down. The wave function can explicitly be expressed as follows:

$$\begin{aligned}
|B\uparrow\rangle &= \left|p:\frac{1}{2}, m_s=\frac{1}{2}\right\rangle \\
&= \left\{\frac{1}{2}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)(udu-duu) + \frac{1}{2}(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)(uud-udu)\right. \\
&\quad \left. +\frac{1}{2}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)(uud-duu)\right\}\frac{\sqrt{2}}{3} \\
&= \frac{2}{3\sqrt{2}}[u(\uparrow)u(\uparrow)d(\downarrow)] - \frac{1}{3\sqrt{2}}[u(\uparrow)u(\downarrow)d(\uparrow)] \\
&\quad -\frac{1}{3\sqrt{2}}[u(\downarrow)u(\uparrow)d(\uparrow)] + \text{permutations (6 more terms)}
\end{aligned}$$

Now calculate the magnetic moment of the proton. The magnetic moment is described with spin operator.

$$\mu = \frac{q}{mc}\vec{S} = \frac{q\hbar}{2mc}$$

The mafnetic moments of up and down quarks are determined by the above relationship. That is:

$$\mu_{\text{up}} = \frac{3}{2}\frac{e\hbar}{2m_{\text{up}}c}, \qquad \mu_{\text{down}} = -\frac{1}{3}\frac{e\hbar}{2m_{\text{down}}c}$$

The magnetic moment is given by

$$\mu_B = \frac{2}{\hbar}\sum_{i=1}^{3}\langle B\uparrow|(\mu_i S_{iz})|B\uparrow\rangle$$

Using the above result, we obtain the magnetic moment for a proton:

$$\begin{aligned}
\mu_p &= 3\Big[\frac{2}{9}(2\mu_{\text{up}} - \mu_{\text{down}}) + \frac{1}{18}\mu_{\text{up}} + \frac{1}{18}\mu_{\text{down}}\Big] \\
&= \frac{1}{3}(4\mu_{\text{up}} - \mu_{\text{down}})
\end{aligned}$$

The magnetic moment of a neutron is given as

$$\mu_n = \frac{1}{3}(\mu_{\text{up}} - 4\mu_{\text{down}})$$

References

[1] D. Griffths, *Introduction to Elementary Particles*. Wiley, 2008