Conservation of Linear Momentum

TA's signature allowing you to take the quiz or leave ____________

Your Name ______________________________

Partners' Names ______________________________

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Obtained reasonable experimental results?	yes ☐
Answered questions?	yes ☐
Cleaned your table?	yes ☐

Do not scratch or write on any equipment.

Introduction

Momentum is a physical quantity which expresses an object moving at a constant velocity:

$$\vec{p} = m\vec{v}$$

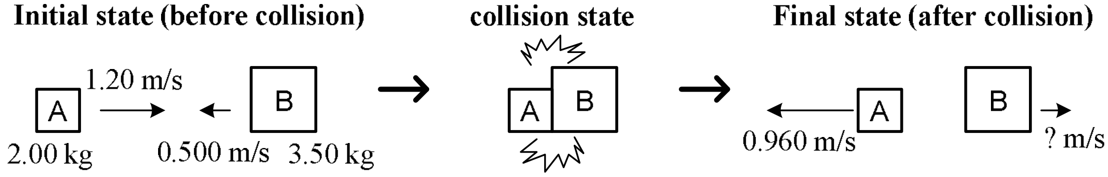
The dimension is [M][L][T^{-1}], so the units are kg·m/s. Momentum has a vector property; namely, the direction also has to be considered. More importantly, momentum is conserved in the event of collision. As you remember the conservation of energy, a conservation law takes two states, which are initial and final states. Suppose there are two objects, and they are realized as one system in terms of collision. The two states are realized as before and after collision. The left- and right-hand sides of the following equation express before (initial) and after (final) of each total momentum, respectively. The equal sign indicates the conservation of linear momentum:

$$\vec{p}_A^{\,\text{ini}} + \vec{p}_B^{\,\text{ini}} = \vec{p}_A^{\,\text{fin}} + \vec{p}_B^{\,\text{fin}}$$

From $p = mv$, each term can be re-written as

$$m_A\vec{v}_A^{\,\text{ini}} + m_B\vec{v}_B^{\,\text{ini}} = m_A\vec{v}_A^{\,\text{fin}} + m_B\vec{v}_B^{\,\text{fin}}$$

Initial state (before collision) **collision state** **Final state (after collision)**



The above figure is an example. It has to be noted that the **sign** of each momentum reflects on the direction of motion. Two objects come from opposite side. Object A is lighter than object B. We suppose that this collision is perfectly elastic. To find the final velocity of object B, conservation of linear momentum can be used. The initial total momentum is obtained from the figure above:

$$\text{Total ini. } \vec{p} = (2.00 \text{ kg})(1.20 \text{ m/s}) + (3.50 \text{ kg})(-0.500 \text{ m/s}) = 0.650 \text{ kg}\cdot\text{m/s}$$

Note that the velocity of object B must be negative 0.500 m/s. The final total momentum can be expressed as

$$\text{Total fin. } \vec{p} = (2.00 \text{ kg})(-0.960 \text{ m/s}) + (3.50 \text{ kg})(\vec{v}_B^{\,\text{fin}}) = -1.92 \text{ kg}\cdot\text{m/s} + (3.50 \text{ kg})(\vec{v}_B^{\,\text{fin}})$$

Conservation of momentum indicates that the initial and final total momenta are equal. Thus,

$$-1.92 \text{ kg}\cdot\text{m/s} + (3.50 \text{ kg})(\vec{v}_B^{\,\text{fin}}) = 0.650 \text{ kg}\cdot\text{m/s}$$

The final velocity of object B can be solved as

$$\vec{v}_B^{\,\text{fin}} = +\,0.734 \text{ m/s}$$

The total momentum is always conserved with a constant velocity; however, the conservation of kinetic energy depends on how they collide. If the collision is perfectly elastic, kinetic energy is conserved. However, if not, such as deformation and cohesion of the objects, the certain amount of energy is lost. This is called inelastic collision.

Objectives:

- To learn and verify conservation of linear momentum
- To see the differences of elastic and inelastic collisions in terms of conservation laws of momentum and energy

Preparation and tips for the experiment:

❶ Start up the software, DataStudio.

❷ Click "Create Experiment."

❸ Click digital channel 1 then select photo gate.

❹ Check the box "Time In Gate, Ch1" shown in the figure.

❺ For the second photo gate, check "Time In Gate, Ch 2" as shown below.

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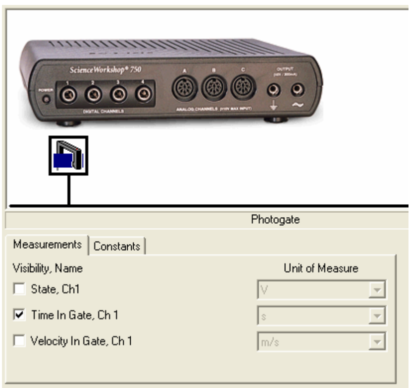
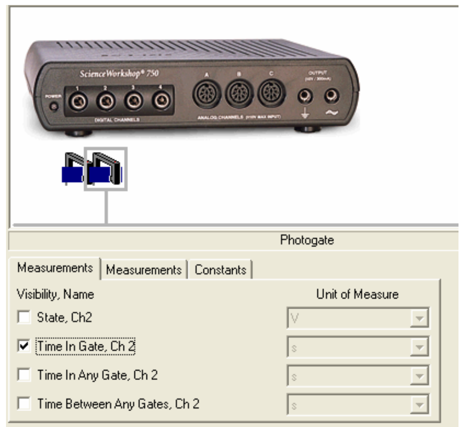
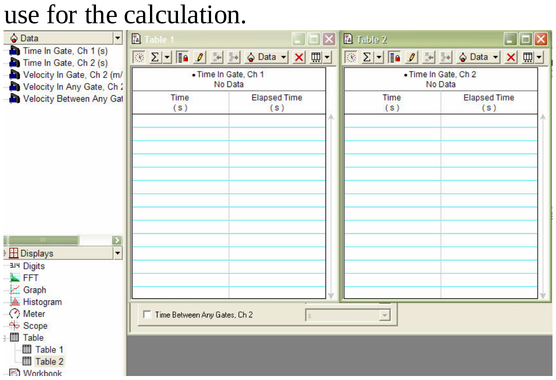
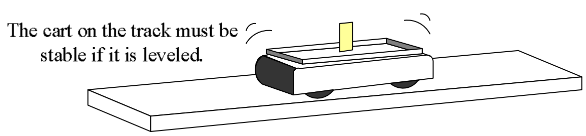




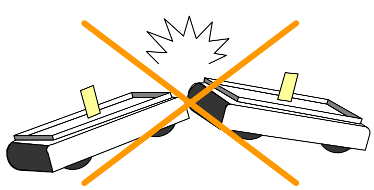
❻ Double click the "Table" in the Display and select "Time In Gate, Ch1 (s)." Again, double click the "Table" and select "Time In Gate, Ch2 (s)" as shown below. (Note: "<u>Elapsed Time</u>" is the one you will use for the calculation.



❼ Put a cart on the track and level it with papers or some object.



❽ The velocity to collide must be constant, which means no acceleration. Push the cart gently. Do not make them collide "impulsively."



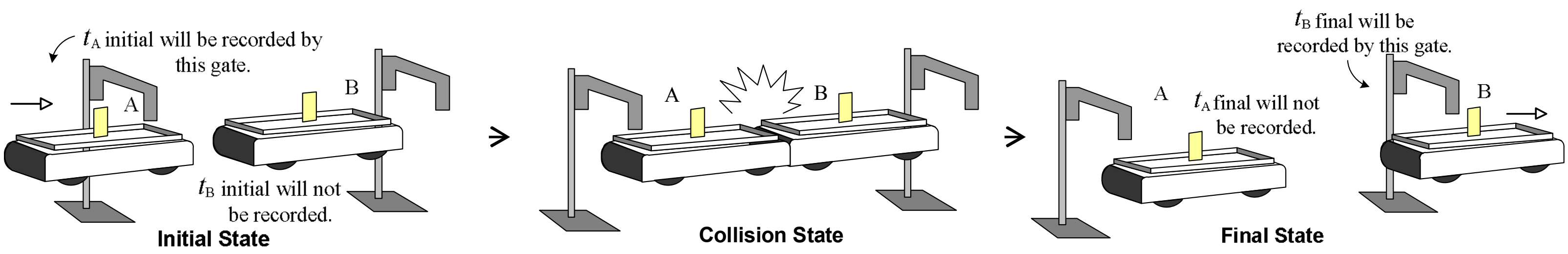
Note: *Through this lab, subscripts A and B represent cart A and cart B, respectively.*

1. Collision of two objects of equal mass (completely elastic collision)

① **See the figure below in the "Initial State."** Place cart B between the two photogates. Cart A is supposed to collide with cart B with a **constant velocity**.

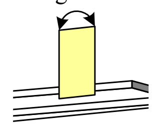
② **Weigh each cart with a balance**. The mass of cart A and the mass of cart B are m_A and m_B, respectively.

Cart masses m_A ______________________ m_B ______________________ (🛈 **The mass of each cart is not necessarily equal.**)



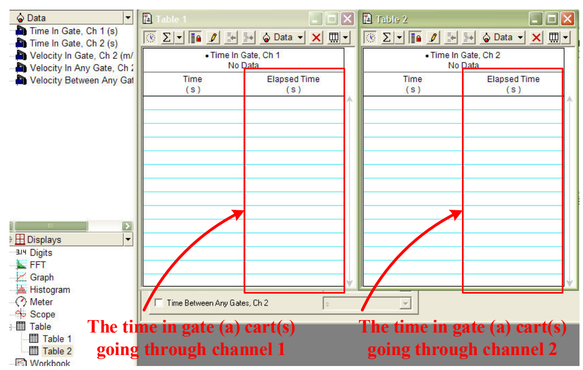
③ To determine the velocity of the cart, we use a piece of tape as a "flag**." Measure the flag width**, ℓ, which is about 1 inch (**Measure it exactly by yourself in meters**.).

Flag widths ℓ_A ______________________ ℓ_B ______________________ (🛈 **Measure the width with a ruler. If the flag made of the masking tape is not aligned, it may be more than 1 inch.**)



④ **Make sure that cart B is still, and push cart A gently against cart B; namely, with a constant velocity. Remember the lab, Intro. to Kinematics to make a constant velocity.** Utilize the magnetic repulsion to make perfectly elastic collision.

⑤ **After the final state in the above figure, read the time in the DataStudio.** Note that you have to pay attention to which cart goes through which gate (Ch 1 or Ch 2). Then, you pick out the time for t_A or t_B / initial or final. In this part, (look at the above figure) suppose the left side of photo gate is Ch 1, cart A goes through Ch 1. Thus, the time appeared in DataStudio for the Ch 1 is "t_A initial" that you will write down in the table below. Similarly, the time recorded in Ch 2 will be "t_B final" for cart B. [The t_A initial or final does not necessarily correspond to Ch 1! Observe carefully how the carts go through each gate.]



Take 3 steps: Record times; calculate velocities; and calculate momenta of carts.

Step 1: ➔ Again, look at the figure above. Time will not be recorded if the object does not move; namely, it is 0 second in the table. Try once to obtain the final result; namely, fill out only trial #1. Make sure if you are doing correctly; then, go to next trials.

Trial #	t_A initial	t_A final	t_B initial	t_B final
1		0	0	
Do not proceed to the next trial unless you obtain a proper result in the first one.				
2		0	0	
3		0	0	

Step 2: ➔ Calculate each velocity of cart with the time recorded above and the flag width. (Velocity = flag width ÷ time above)

Trial #	v_A initial	v_A final	v_B initial	v_B final
1				
Do not proceed to the next trial unless you obtain a proper result in the first one.				
2				
3				

Step 3: ➔ Calculate each momentum of cart with each cart mass and corresponding velocity above. Then, compare the total momentum of initial and final states to see if the momentum is conserved. (Momentum = mass × velocity)

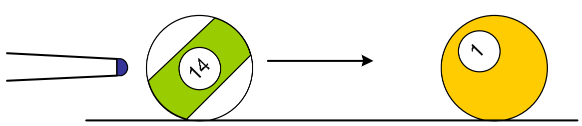
Trial #	p_A initial	p_A final	p_B initial	p_B final	Total initial momentum p_A ini + p_B ini	Total final momentum p_A fin + p_B fin	% diff. of the last two columns
1							
Do not proceed to the next trial unless you obtain a proper result in the first one.							
2							
3							

Find the percentage difference $\frac{|\text{Total ini. } p - \text{Total fin. } p|}{(\text{Total ini. } p + \text{Total fin. } p)/2} \times 100 \ (\%)$ for each value.

★ **The momentum of the entire system should be conserved.** Momentum of a mass moving at velocity v is $\boldsymbol{p} = m\boldsymbol{v}$. In this part, 100% of cart A's momentum p_1 is supposed to be transferred to B.

Question for part 1: How well does your experiment explain conservation of the momentum?

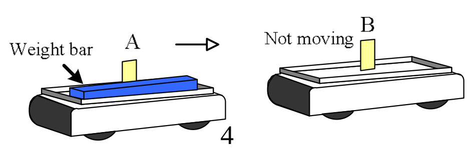
Conceptual question:
As shown in the figure, when the billiard ball that you hit collides with another billiard ball with a constant velocity, what will it happen? (This is exactly the same situation as above experiment.)



2. Collision of two objects of unequal mass (completely elastic collision)

- Heavy hits light

① Place cart B between the two photogates. Push the cart A against the cart B. Let them collide by the magnetic repulsion. Notice that this time the cart A will **not** stop after the collision, so you need to determine the final velocity of cart A as well.



② The momentum of this system is supposed to be conserved.
Draw a simple diagram as shown in the first part. **[Try it first to see how carts are moving.]**

Indicate how the initial times are recorded.		Indicate how the final times are recorded.
Initial State	**Collision State**	**Final State**

Cart masses m_A ______________________ m_B ______________________

Flag widths ℓ_A ______________________ ℓ_B ______________________

Measure the time with photo gates as illustrated above. Time will be zero if the object does not have motion.

Trial #	t_A initial	t_A final	t_B initial	t_B final
heavy hits light				

Calculate each velocity with the time measured above and the flag width.

Trial #	v_A initial	v_A final	v_B initial	v_B final
heavy hits light				

Calculate each momentum with each cart mass and corresponding velocity. Then, compare the total momentum of initial and final states to see if the momentum is conserved.

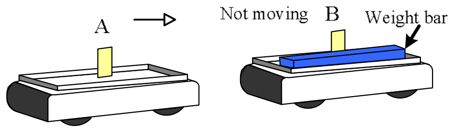
Trial #	p_A initial	p_A final	p_B initial	p_B final	Total initial momentum p_A ini + p_B ini	Total final momentum p_A fin + p_B fin	% diff. of the last two columns
heavy hits light							

Find the percentage difference $\frac{|\text{Total ini. } p - \text{Total fin. } p|}{(\text{Total ini. } p + \text{Total fin. } p)/2} \times 100\,(\%)$ for each trial.

Question for part 2: How well does your experiment explain conservation of the momentum?

- <u>Light hits heavy</u>

① Place cart B between the two photo gates. Push the cart A against the cart B. Let them collide <u>by the magnetic repulsion</u>. Notice that this time the cart A will recoil after collision, so you need to determine the final velocity of cart A as well.



② The momentum of this system is supposed to be conserved. (<u>Note: Momentum is a vector. In this one-dimensional problem, if the velocities are in different directions, be sure to use appropriate signs</u>.)

Draw a simple diagram as shown in the first part. **[Try it first to see how carts are moving.]**

Indicate how the initial times are recorded.		Indicate how the final times are recorded.
Initial State	**Collision State**	**Final State**

Cart mass m_A ______________________ m_B ______________________

Flag width ℓ_A ______________________ Flag width ℓ_B ______________________

Measure the time with photo gates as illustrated above. Time will be zero if the object does not have motion.

Trial #	t_A initial	t_A final	t_B initial	t_B final
light hits heavy				

Calculate each velocity with the time measured above and the flag width.

Trial #	v_A initial	v_A final	v_B initial	v_B final
light hits heavy				

Calculate each momentum with each cart mass and corresponding velocity. Then, compare the total momentum of initial and final states to see if the momentum is conserved.

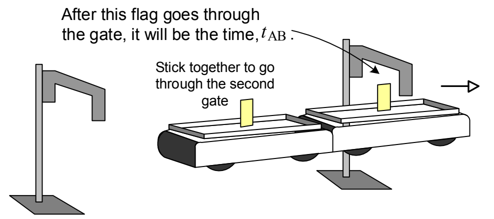
Trial #	p_A initial	p_A final	p_B initial	p_B final	Total initial momentum p_A ini + p_B ini	Total final momentum p_A fin + p_B fin	% diff. of the last two columns
light hits heavy							

Find the percentage difference $\dfrac{|\text{Total ini. } p - \text{Total fin. } p|}{(\text{Total ini. } p + \text{Total fin. } p)/2} \times 100\,(\%)$ for each trial.

Question for part 2: How well does your experiment explain conservation of the momentum? [This part may have a large error. Discuss the causes of error with your partner or TA.]

3. Completely inelastic collision

Note:



① Place cart B between the two photogates. Push the cart A against the cart B with a constant velocity.

② Let them collide with Velcro so after collision they stick together. (Although the collision is completely inelastic, the total momentum is still conserved. However, **about 50%** of the kinetic energy is lost, if the two masses are equal.)

Attention! During any collision, the system's total momentum is always conserved with **constant velocity**. **Kinetic energy is conserved only if the collision is elastic.** For inelastic collisions, part of kinetic energy is lost due to converted to other forms of energy.

Draw a simple diagram as shown in the first part. **[Try it first to see how carts are moving.]**

Indicate how the initial times are recorded.

Indicate how the final times are recorded.

Initial State **Collision State** **Final State**

Cart masses m_A ______________ m_B ______________

Flag widths ℓ_A ______________ ℓ_B ______________

The time, *t*, is obtained by the measurement. The velocity and momentum, *v* and *p*, are obtained by calculation. The suffix, "AB", indicates that carts A and B are stuck together. See the figure above.

Trial #	t_A (initial)	t_{AB} (final)	v_A (initial)	v_{AB} (final)	p_A (Total initial)	p_{AB} (Total final)	% difference $\frac{\lvert p_A - p_{AB}\rvert}{(p_A + p_{AB})/2}\times 100$
1							
2							

Question for part 3: How well does your experiment explain conservation of the momentum?

Calculate the kinetic energy for initial and final states. Use the values from the above table.

Trial #	KE_A $= \frac{1}{2} m_A v_A^2$ (initial)	KE_{AB} $= \frac{1}{2} (m_A + m_B) v_{AB}^2$ (final)	% difference $\frac{\lvert KE_A - KE_{AB} \rvert}{(KE_A + KE_{AB})/2} \times 100$
1			
2			

Question for part 3: How does the energy change? Is the energy conserved or lost? (Note that the large percentage difference of this part does not illustrate simple scientific errors.)
