Moments of Inertia
(Rotational Inertia)

Your Name _____

Partners' Names _____

_____

TA's signature allowing you
to take the quiz or leave _____

Obtained reasonable experimental results?	yes ☐
Answered questions?	yes ☐
Cleaned your table?	yes ☐

This lab is case sensitive. Please do NOT mix up *R* and *r*, and *M* and *m*.

Introduction

Rotational motions involve more than linear motions. Inertia gives us an idea of how hard or easy the object is accelerated. The more inertia it has, the harder it is accelerated and decelerated. In the linear motion, inertia corresponds to its mass as a point object. In practice, an object has an extended shape as a three dimensional sense. Especially, when the object rotates, the shape plays a more important role and it needs more description for the inertia besides its mass. This is called rotational inertia or moment of inertia. The moment of inertia is differentiated by the mass distribution of the object. Simply put, if the shape is different, the "rotational" inertia will be different even though the masses are equal. It is proportional to the mass and radius squared. The dimension of moment of inertia is $[M][L^2]$, so the unit is kg·m^2. For example, if the object is a circular disk, the moment of inertia is

$$I_{disk} = \frac{1}{2}MR^2$$

where *M* is the mass and *R* is the radius of the object. As stated, this affects how it is accelerated in rotational motion. The moment of inertia of a ring object is different:

$$I_{ring} = MR^2$$

From the above, under the same conditions (equal mass and radius), the ring has more inertia than that of disk.

In experiment, the moment of inertia of an object can be obtained by the time it is accelerated by a hanging mass. In addition, the base apparatus also has moment of inertia; therefore, it has to be subtracted from the final result since the moment of inertia from the experiment is combined by the apparatus and an object.

Objectives:
- To learn the physical significance of rotational inertia
- To test the difference of rotational acceleration with two objects (disk and ring) due to different moments of inertia

1. Moment of Inertia of Apparatus

The base of the apparatus has a moment of inertia. That is I_A. The moment of inertia is a little complicated and it should be found by an experiment.
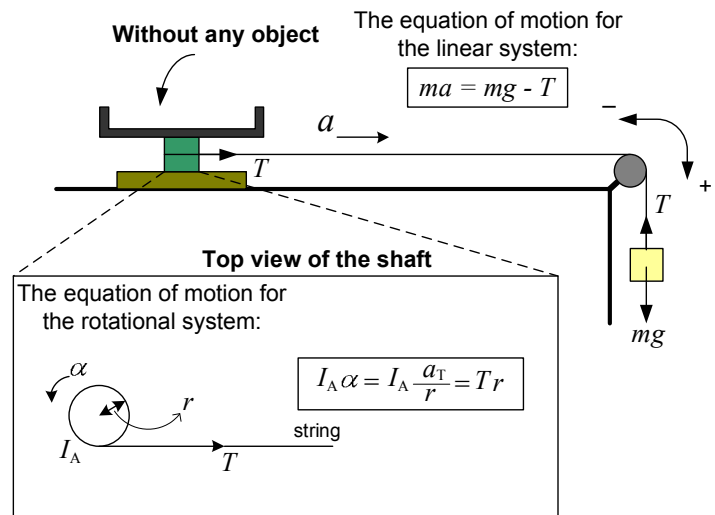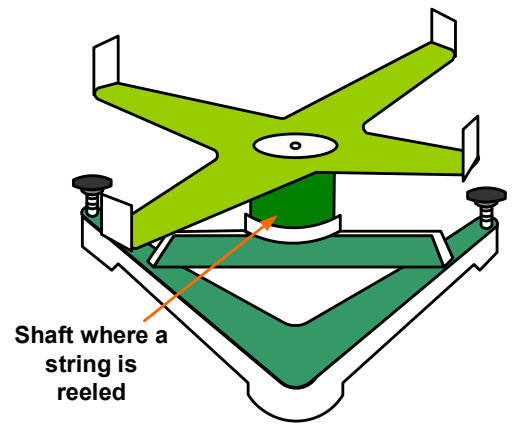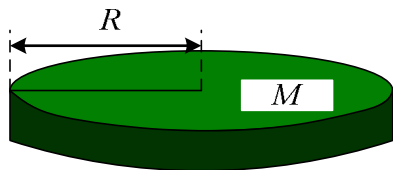
I_A can be calculated from two equations of motion (linear and rotational systems) as shown in the figure. Set up the simultaneous equations and solve for I_A. (Note: T times r is the torque associated with the shaft.)

Shaft where a string is reeled

$$\begin{cases} ma_T = mg - T \\ I_A \dfrac{a_T}{r} = Tr \end{cases}$$

$$\rightarrow I_A = mr^2 (g - a_T)/a_T$$

This moment of inertia will be subtracted in the later parts to obtain the expected one.

To obtain the moment of inertia of the ring or disk object, measure the diameter and divide it by two to get the radius, R. The mass, M, is written on the object.

Without any object

The equation of motion for the linear system:

$$ma = mg - T$$

Top view of the shaft

The equation of motion for the rotational system:

$$I_A \alpha = I_A \frac{a_T}{r} = Tr$$

string

I_A T

a

T

mg

R

M

STOP!
Make sure following variables. Describe each by your own words. [If you use the value of M for m, you will have a totally different result.]

m _____ ; M _____

r _____ ; R _____

Procedure
① **Find the shaft diameter; then calculate the radius.**
Use a caliper and measure the diameter of the shaft. Take three significant figures (ex. 0.0467 m). Dividing it by 2, you will obtain the radius, r.

② **Measure the distance between photo gates.**

③ **Take a sufficient length (1 meter) of string, and reel it around the shaft.**

With/without object

④ **Make sure the position of each device.**
The Pulley, photo gates, and the apparatus should be located properly. The string from the apparatus should be <u>parallel</u> with pulley, and the apparatus must be placed on far from the pulley. Check with your TA.

⑤ **Hook a hanging mass at the other end of string; then record the falling time, t, with DataStudio.**

h
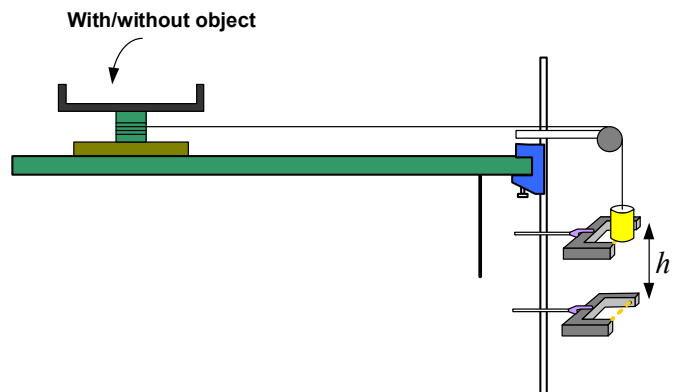
The hanging mass is fixed as 0.2 kg (200 g). <u>The initial point must be as close to the first gate as possible before the release.</u>

<u>**Note**: The string can get caught by the photo gate. Please adjust the position of the pulley and cut extra string from the knot to prevent this.</u>

⑥ **Calculate the acceleration and the moment of inertial of the shaft.**

Assuming that the acceleration is constant, we can make use of $\overline{v} = \frac{v_f + v_i}{2}$ to obtain the average velocity.

Since the initial velocity must be zero, $v_i = 0$, and the another expression of the average velocity is $\overline{v} = \frac{h}{t}$,

so the final velocity is $v_f = \frac{2h}{t}$. Therefore, the acceleration reaching the final velocity is $a = \frac{2h}{t^2}$.

- **Object 0: Only the apparatus**

Shaft diameter D =_____ (m)   Note: 1.0 cm → 0.01 m; 1.0 mm → 0.001 m

Shaft radius r = D/2 = _____(m)

Distance between photo gates, h = _____(m)

Hanging mass, m = _____0.200_____ (kg)

Gravitational acceleration, g = _____9.807_____ (m / s^2)

Trial #	t	$a = 2h/t^2$	$I_A = m r^2 (g - a) / a$
1			
2			
3			
4			

Average and standard deviation, I_A = (±   ) (kg·m^2)
<u>[This will be used in the next parts.]</u>

2. **Moments of Inertia of Various Objects**

① **Measure the diameter of the objects** (Disk or Ring), **and obtain the radius.**
The measurement should be taken three times by different people, and obtain the average.

② **Calculate the theoretical values of moments of inertia** (Disk or Ring)**.**
Use the labeled mass to calculate them.

③ **Record the falling time intervals for three times** (Disk or Ring)**.**
You can just measure the intervals for three times consecutively then obtain the average. If you get a result very off, take another data. Then, calculate the average from the three data.

④ **Obtain the experimental values for the moments of inertia** (Disk or Ring)**.**
<u>Do not get confused with the variables, r and R.</u> You have to use 'r', which is shaft radius, to calculate these. The value, I_A, is from part 1.
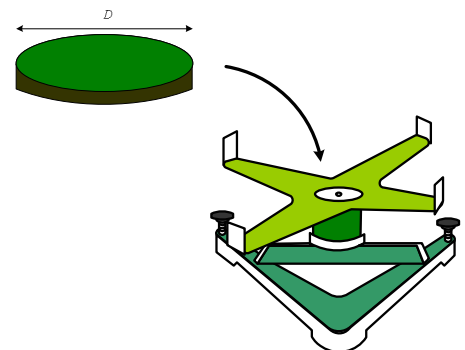
- Preparation

Measure the diameter of the disk three times by different people and calculate the average.

Diameter
$\left\{\begin{array}{l}\text{_____ ( ) } \leftarrow \text{units} \\ \text{_____ ( ) } \leftarrow \text{units} \\ \text{_____ ( ) } \leftarrow \text{units}\end{array}\right.$ Average diameter _____( )

Radius (Disk) R =Average diameter÷2_____ ( ) ← units

Measure the diameter of the ring three times by different people and calculate the average.

Diameter
$\left\{\begin{array}{l}\text{_____ ( ) } \leftarrow \text{units} \\ \text{_____ ( ) } \leftarrow \text{units} \\ \text{_____ ( ) } \leftarrow \text{units}\end{array}\right.$ Average diameter _____( )

Radius (Ring) R =Average diameter÷2_____ ( ) ← units

- **Object 1: Disk**

➢Theoretical value of the moment of inertia

The labeled mass M_____ ( ) ← units

Theoretical $I_{disk} = \frac{1}{2} M R^2 =$ _____ ( )←units **(1)**
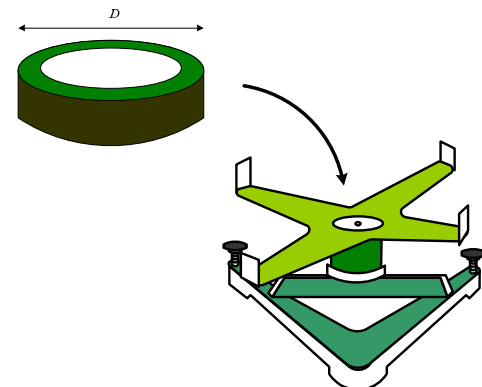
➢Experimental value of the moment of inertia

Falling times
$\left\{\begin{array}{l}\text{_____( ) } \leftarrow \text{units} \\ \text{_____( ) } \leftarrow \text{units} \\ \text{_____( ) } \leftarrow \text{units}\end{array}\right.$

Note: I_{total} is the combination of I_{disk} and I_{shaft}.

Average t	$a = 2h/t^2$	$I_{total} = m\,r^2\,(g - a)\,/\,a$
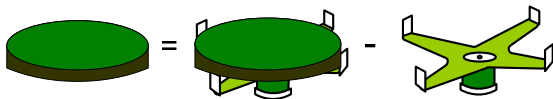
Experimental $I_{disk} = I_{total} - I_A =$ _____ ( )←units **(2)**

Question:
How well does the measured moment of inertia agree with the calculated (theoretical) one [**(1)** & **(2)**] including the uncertainty of I_A? (It should not be more than 10 % error with taking the standard deviation into account.)

- ## **Object 2: Ring**

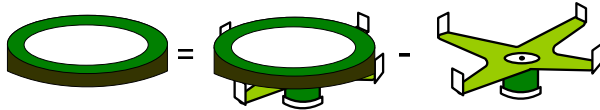➢Theoretical value of the moment of inertia

The labeled mass M_____ (   ) ← units

Theoretical $I_{ring} = M R^2$ =   = _____ (   ) ← units **(3)**

➢Experimental value of the moment of inertia

$$\text{Falling times} \begin{cases} \underline{\hspace{4cm}}(\ \ ) \leftarrow \text{units} \\ \underline{\hspace{4cm}}(\ \ ) \leftarrow \text{units} \\ \underline{\hspace{4cm}}(\ \ ) \leftarrow \text{units} \end{cases}$$

| Average t	$a = 2h/t^2$	$I_{total} = m\,r^2\,(g - a)\,/\,a$

Experimental $I_{ring} = I_{total} - I_A =$ _____ (   ) ← units **(4)**

Question:
The experimental value in **(4)** is supposed to agree with the theoretical value in **(3)**. However, you likely obtain a smaller value in experiment than that in theory (The theoretical value is always bigger!). Discuss it with the following question: Is the shape of object (Ring) exactly matched with the theoretically ideal object? (***Please go to next section!***)

❋ If the thickness of the ring is very thin, you can use MR^2 for the moment of inertia. However, there is a certain thickness, you have to use following formula:
(Note: R_a and R_b are the radiuses of the outer and inner circles.)
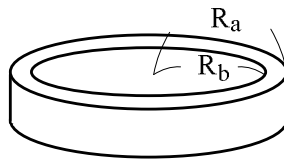
$$I = \frac{1}{2}\left(R_a^2 + R_b^2\right)M$$

Now, measure R_a and R_b, then calculate the above moment of inertia.

R_a _____ (   ) ← units; R_b _____ (   ) ← units;

Labeled Mass _____ (   ) ← units

$$I = \frac{1}{2}\left(R_a^2 + R_b^2\right)M =$$ _____ (   ) ← units **(5)**
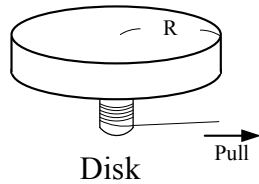
Question:

How well does the measured moment of inertia agree with the calculated (theoretical) one [(**4**) & (**5**)]?
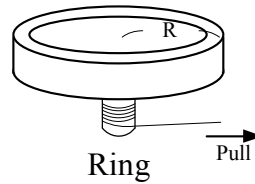
Conceptual Activity

Pull the string by your hand <u>slowly</u> as follows. (<u>Be careful. The string may cut your fingers.</u>) Remember how it feels.

mass = M   mass = M

Disk   Pull   Ring   Pull

Question:

Which one is harder to accelerate, ring or disk? Why does this happen even though both have about equal mass? Explain the reason in terms of moment of inertia. Discuss this with your lab partners, and make sure with your TA.