

Newton's Laws and Equation of Motion

TA's signature allowing you
to take the quiz or leave _____

Your Name _____

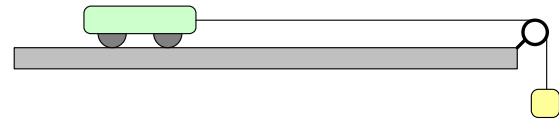
Partners' Names _____

Obtained reasonable experimental results?	yes <input type="checkbox"/>
Answered questions?	yes <input type="checkbox"/>
Cleaned your table?	yes <input type="checkbox"/>

Introduction

Newton's laws consist of "the law of inertia", "the law of motion", and "the law of action and reaction." The first law depicts that if there is no external effect, an object must be still or moving at a constant velocity, which leads to a concept of the net force. The second law quantifies the force in terms of acceleration and mass of the object. The third law illustrates the existence of the counter force which is related to normal forces and tension, etc. Solving the equation of motion involves these three principles, and we have the general expression, $\Sigma F=ma$, (The net force on one object with one direction is equal to the mass of the object times its acceleration based on the physical conditions.), for each object and each axis. This also needs consideration of the vector properties of forces. The dimension of force is $[M][L][T^{-2}]$. The SI unit of force is newtons (N).

For this lab, Atwood's machine is used to verify the Newton's laws. The general definition of Atwood's machine is a uniformly accelerated motion of two objects connected by a string suspended over a pulley as shown.



Let us divide the above figure into each object. For object 1 (cart) in the moving direction, the net force is only the tension as shown. For object 2 (hanging mass) in the moving direction, the net force is taken from the gravitational force and the oppositely directed tension. The figure shows that object 1 has tension as the net force in the positive direction. The equation of motion becomes:

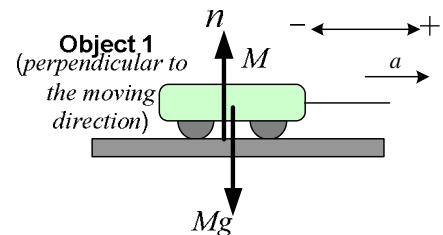
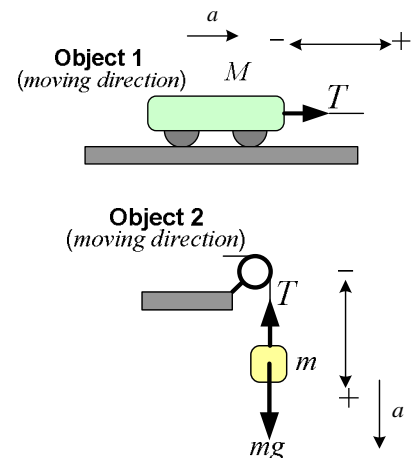
$$\Sigma F_1 = +T = Ma$$

The net force for object 2 is a gravitational force, mg (positive direction), and a tension (negative direction). Therefore,

$$\Sigma F_2 = +mg - T = ma$$

Then, consider another independent direction which is perpendicular to the moving axis.

For perpendicular to the moving direction of object 1, the net force is the gravitational force plus the normal force as shown. However, there is no motion in this direction; namely, there is no acceleration in the perpendicular direction. Thus, the net force is equal to zero. For object 2, it does not have external forces in horizontal direction (zero net force).



According to Newton's insight, each net force is equal to the mass times the acceleration, $\Sigma F = ma$. The acceleration for both objects is supposed to be equal because they are connected by a string.

Objectives:

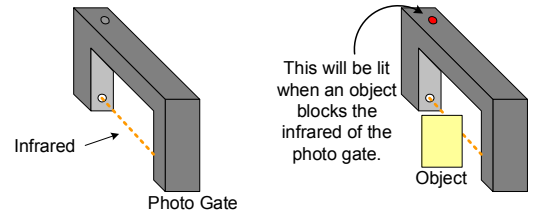
- To learn the free-body diagram to find equations of motion
- To master how Newton's laws are conceptualized in the accelerated motions
- To verify the experimental accelerations as equal to the theoretical ones

Think about it before doing the experiment:

From the introduction, convince yourself or your friends how to obtain the equations of motion in the next two sections. List the basic procedure in your mind.

🔗 **Learn about the equipment!** (Before conducting the experiments)

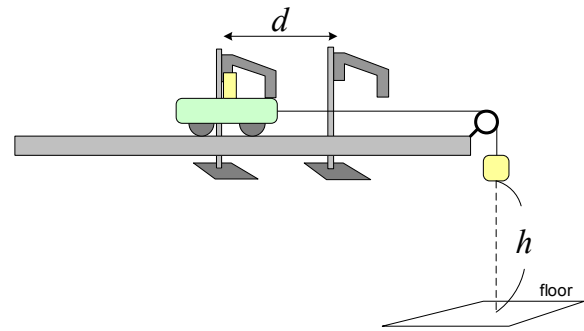
- A photo gate uses infrared to detect an object passing. When an object passes it, the light on the gate will be turned on. Do you understand the mechanism of the photo gate? Yes ___ No ___
- The formula assumes that **the initial velocity is zero**; therefore, the cart with a flag must be set as close to the first gate as possible.



<p>① Set up two photo gates and a cart with a flag as follows. (Top view)</p>	<p>② Make the cart enter the first photo gate slowly until it flashes.</p>	<p>③ Pull it back a little bit, then hold the cart when the light is just turned off. Click start and release the cart.</p>
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Do you understand that it is very important to obtain an accurate result? Yes ___ No ___

- The cart must be uniformly accelerated through the two gates. Namely, the falling distance has to be longer than the distance between gates. The h has to be greater than d . Do you understand when h is shorter than d , the result will be completely deviated from the expected value? Yes ___ No ___ (If no, consult your TA.)



1. In case of the leveled plain

The equation of motion:

“moving”-axis:

$$\left\{ \begin{aligned} \sum F_1 = T = Ma & \quad (1) \\ \sum F_2 = mg - T = ma & \quad (2) \end{aligned} \right.$$

$$\sum F_2 = mg - T = ma \quad (2)$$

perpendicular to the moving-axis:

$$\sum F_1 = n - Mg = 0 \quad (\text{This is not used})$$

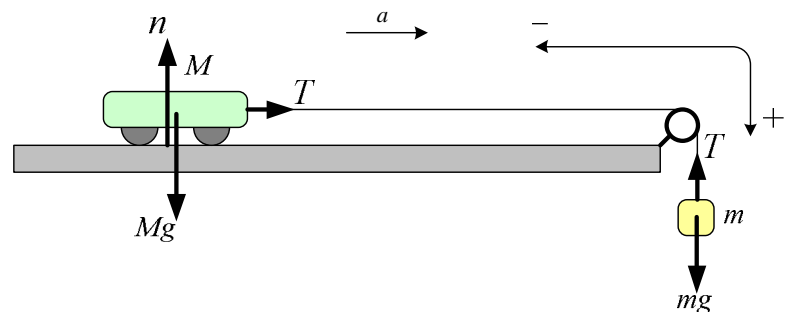
since there is no friction.)

Adding (1) to (2), we obtain

$$mg = Ma + ma$$

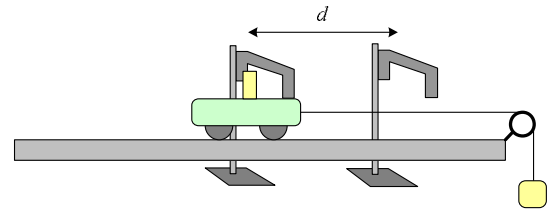
Solve for a .

$$a = \frac{mg}{M + m} \quad (\text{This is the theoretical formula to predict the acceleration.})$$



Procedure

- ⑥ Set up the equipment as shown in the picture and measure distance between photo gates, d .
- ⑦ Start up DataStudio. Click “Creat Experiment.”
- ⑧ Click on the interface (digital channel 1). Select “Photo Gate.”
- ⑨ Click on the interface (digital channel 2). Select “Photo Gate.”
- ⑩ Display “Table.” Select “Time Between Any Gates.”
- ⑪ Record the “elapsed time.”



Gravitational acceleration : _____ 9.81 _____ (m/s²)

Distance, d : _____ () ⇔ unit

Case #	Mass of the cart, M (kg)	Mass of the hanging weight, m (kg)	t (s)	$a = \frac{2d}{t^2}$ [From the experiment] (m/s ²)	a [Predicted] (m/s ²) Find this equation from this manual.	% Difference (b/w predicted and experimental)
1		0.050				
2		0.100				
3		0.150				
4		0.200				

◆ **Think about this before you ask your TA:** An experimental value is obtained by your measurement. The theoretical value is calculated based upon the related equation.

$$\% \text{ difference} = \frac{|a_{\text{exp}} - a_{\text{predicted}}|}{\frac{1}{2}(a_{\text{exp}} + a_{\text{predicted}})} \times 100 \%$$

2. In case of the inclined plain

The equation of motion:

“moving”-axis:

$$\begin{cases} \sum F_1 = Mg \sin \theta - T = Ma & (1) \\ \sum F_2 = T - mg = ma & (2) \end{cases}$$

perpendicular to the moving-axis:

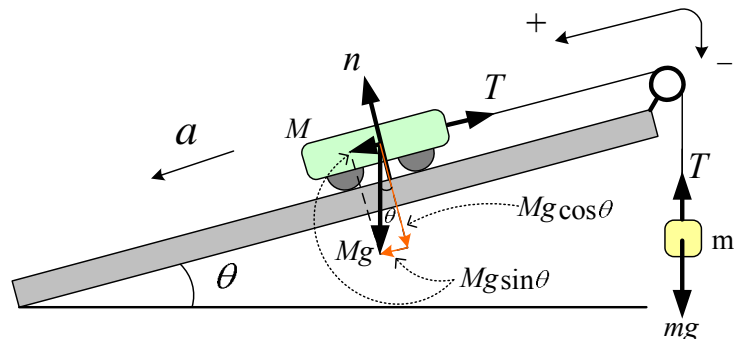
$$\sum F_1 = n - Mg \cos \theta = 0 \quad (\text{This is not used since there is no friction.})$$

Adding (1) to (2), we obtain

$$Mg \sin \theta - mg = Ma + ma$$

Solve for a .

$$a = \frac{Mg \sin \theta - mg}{M + m} \quad (\text{This is the theoretical formula to predict the acceleration.})$$

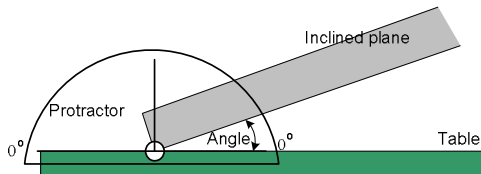


☛ **Check these out!**

Look at the figures to exercise safety first.

Pull the precision track to the edge of the table. Place the center of a protractor as follows. Read the angle from the horizontal to the bottom of the track.

Note: The angle will be changed when the track moves horizontally!



Similarly to the first part, measure the distance between photo gates, d .

Then, measure the elapsed time, t , for each case.

* Select the hanging mass so the cart can be accelerated downward on the track.

The angle, θ : _____

Gravitational acceleration: 9.81 (m/s²); Distance, d : _____ () ⇔ unit

Case #	Mass of the cart, M (kg)	Mass of the hanging weight, m (kg)	t (s)	$a = \frac{2d}{t^2}$ [From the experiment] (m/s ²)	a [Predicted] (m/s ²) Find this equation from this manual.	% Difference (b/w predicted and experimental)
1		0.020				
2		0.030				
3		0.040				
4		0.050				

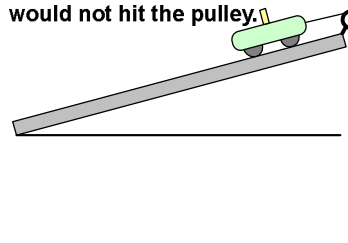
$$\% \text{ difference} = \frac{|a_{\text{exp}} - a_{\text{predicted}}|}{\frac{1}{2}(a_{\text{exp}} + a_{\text{predicted}})} \times 100 \%$$

Conclusion and Discussion (Address the following questions to your report.)

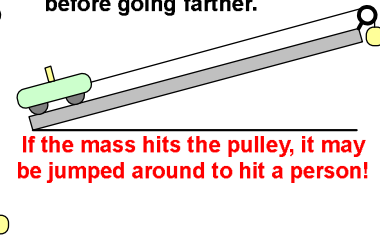
1. Are all of your predicted acceleration values within 10 % error?

2. What factors do you think may cause the experimental value to be different from the predicted value? **Do not mention human error!**

Take an enough length of string so the hanging mass would not hit the pulley.

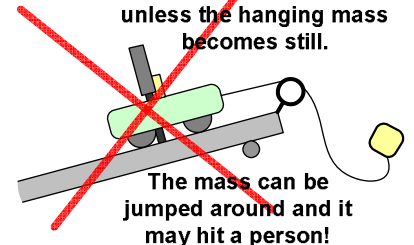


Try to catch the cart before going farther.

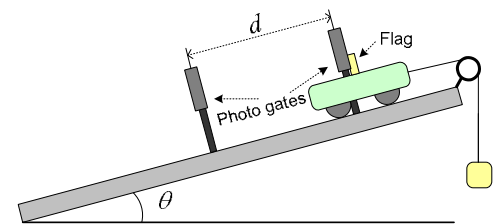


If the mass hits the pulley, it may be jumped around to hit a person!

Do not release the cart unless the hanging mass becomes still.

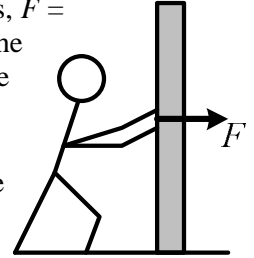


The mass can be jumped around and it may hit a person!



Questions to discuss in the lab

1. In the second part of the experiment, if the hanging mass is going down to ground, how will the acceleration be? (*Hint: the form of the equation is almost the same as original except one of the signs, + or -.*)
2. Human cannot usually move a sturdy wall by pushing. Newton's second law says, $F = ma$. The wall is not in motion and it gives that the acceleration, a , is zero. From the second law, the force will be calculated as zero. This seems contradicted since the force exerted to the wall is not zero... How do you explain the entire discussion? We have used the description of motion with $\Sigma F = ma$ instead of $F = ma$. The expression, ΣF , is the net force of the system that is the sum of every force on the specific object. In this example, the reason why the wall is not in motion by pushing is because other force effectively cancels it as the "net" force on the wall.



Think about how important the concept of the net force is. Why do you have to use the normal force and tension to derive equations of motion?

Advanced Note:

The action-reaction forces exert two different objects; thus, these cannot be cancelled in terms of the net force. However, the gravitational force, mg , and the normal force, n , are not the action-reaction force since both forces exert the same object. The normal force is a derived force from which the object pushes against the table due to gravity based upon the principle of action-reaction force. The same discussion can be done with tension.