Derivation of the final velocity of rolling objects

<u>Preparation</u>

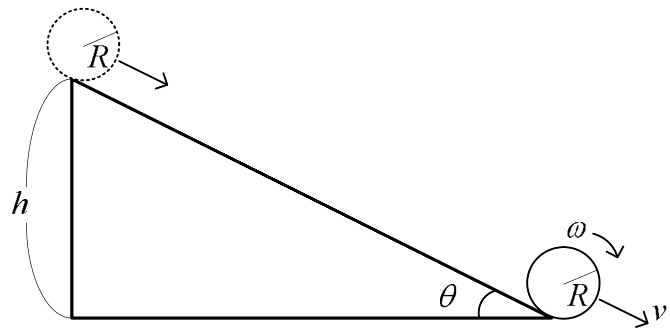
The moments of inertia:

Solid disk $I_D = \frac{1}{2}MR^2$ (1)

Sphere $I_S = \frac{2}{5}MR^2$ (2)

Ring $I_R = MR^2$ (3)

where M is the mass, and R is the radius of the object.



From the figure, we can set up an equation by considering the energy conservation, (potential energy) = (translational kinetic energy) + (rotational kinetic energy)

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \quad (4)$$

where ω is the angular speed. Each object of the moment of inertia, I, is different. To find the final velocity, v, plug each one of (1), (2) or (3) into (4), and solve for v. Note that $\omega = \frac{v}{R}$.

For a disk,

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\cdot\frac{1}{2}MR^2 \cdot \frac{v^2}{R^2}$$

$$Mgh = \frac{3}{4}Mv^2$$

$$\therefore v = \sqrt{\frac{4}{3}gh} \quad (5)$$

For a sphere,

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\cdot\frac{2}{5}MR^2 \cdot \frac{v^2}{R^2}$$

$$Mgh = \frac{7}{10}Mv^2$$

$$\therefore v = \sqrt{\frac{10}{7}gh} \quad (6)$$

For a ring,

$$Mgh = \frac{1}{2}Mv^2 \frac{1}{2}MR^2 \cdot \frac{v^2}{R^2}$$

$$Mgh = Mv^2$$

$$\therefore v = \sqrt{gh} \quad (7)$$