## Thin Lenses and its Equations



The object lamp might be hot. Exercise caution while using the lamp.

## Introduction

Thin lenses can be formulated by the relation between the object distance, the image distance, and the focal length. This is derived from the geometrical properties of the ray tracing diagram, and the equation is called the thin lens equation:

$$
\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}
$$

where $s, s^{\prime}$, and $f$ are the object distance, the image distance, and the focal length, respectively. The sign of focal length depends on the type of lenses. This equation holds both convex (converging) and concave (diverging) lenses.

The image of an object through a lens can look large or small that is called magnification. This also characterizes the lens property. The definition of magnification, $M$, comes from the ratio between the object height, $h$, and its image height, $h^{\prime}$ :

$$
M=\frac{h^{\prime}}{h}
$$

When the image reduced, the magnification will be less than 1 . When the image magnified, the magnification will be more than 1 . From the geometrical property of the ray diagram, the magnification can also be expressed with the object and image distances:

$$
M=-\frac{s^{\prime}}{s}
$$

Note that the positive and negative magnifications indicate that the image is inverted and upright, respectively. In the measurement, if the image is found to be inverted, the image height, $h^{\prime}$, must be expressed as negative. For two lenses, you can use the above expressions twice to find the final focal length and its magnification.

## Objectives:

- To verify the thin lens equation and the magnification equation experimentally
- To learn how to find the focal length of a concave lens using the thin lens equation (combination of thin lenses)


## 1. The focal length and magnification of a converging lens:



Before you determine the position of the lens, move the lens back and forth to obtain the sharpest image.
(4) After obtaining the image and object distances, calculate the focal length with the thin lens equation.
Thin lens equation: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s^{\prime}}$. The focal length can be solved as $f=\frac{s \cdot s^{\prime}}{s+s^{\prime}}$.

Table 1

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Object distance $s$ | Image distance $s^{\prime}$ | Focal length $f$ | Object height $h$ | $\begin{gathered} \text { Image height } \\ h^{\prime} \\ \text { (Put a negative } \\ \text { sign if the image is } \\ \text { inverted.) } \\ \hline \end{gathered}$ | Magnification $M=h^{\prime} / h$ (direct) | Magnification $M=-s^{\prime} / s$ <br> (distance ratio) | \{magnified or reduced\}, \{upright or inverted \} |
| Try less than 20 cm |  |  |  |  |  |  |  |
| $\Uparrow$ Did you ob | ny image o | screen? Yo | re NOT supp | sed to get any i | ge. Discuss th | conceptual q | ons below |
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Question: Does the experimental result of focal lengths for convex (converging) lens correspond to the reference value $(+20.0 \mathrm{~cm}$ or $+0.200 \mathrm{~m})$ within the uncertainty?

Question: In columns 6 and 7, are the measurements of magnifications consistent between the direct measurement and the distance ratio?
$\square$

## Conceptual questions:

(1) When you put the convex lens in front of light source (object) where the object distance is smaller than the focal length $(20 \mathrm{~cm})$, do you obtain any image on the screen?
(2) When the object distance is lager than the focal length $(20 \mathrm{~cm})$, do you find the image on the screen?
[Hints: This is related to real and virtual images obtained with a convex lens. Ask your TA to discuss this.]
(3) Does the accuracy of the focal length obtained from the experiment above depend on the object distance? How does it depend on?

## 2. Finding the focal length of a diverging lens (using two lenses):

(Purposes of this part) If you make the same experimentation as above with a concave lens, it is not possible to obtain the focal length of the lens since a concave lens does not have any image focused on a screen. To obtain the focal length of a concave lens, you also have to use a convex lens. It is also pedagogical to learn the procedure to deal with two lenses with the thin-lens and magnification equations.

- First focus an image only with the converging (convex) lens.
Write down the object and image distances. Also measure the magnification. (Do not put this convex lens within its focal length. The lens should be placed at between 35 cm and 55 cm . Do not move this lens after you determined.)


Object distance $s_{1}$ : $\qquad$ Image distance $s_{1}^{\prime}$ : $\qquad$

* Magnification of the converging lens only

Direct measurement $\rightarrow M_{1}=\frac{\mathrm{h}_{1}^{\prime}}{\mathrm{h}_{1}}$ $\qquad$ (If the image is inverted, h' will be negative.)

Distance ratio $\rightarrow-s_{1}^{\prime} / s_{1}$ : $\qquad$

- Insert the diverging (concave) lens between the convex lens and screen.
The separation between two lenses should not be too large. That makes the result inaccurate.
- By adjusting the screen, focus the image. NEVER move the convex lens! Follow the data sheet to record the measurements.


Note: The focused image with both lenses should not be focused with only the convex lens. Namely, after you obtained an image with two lenses, take the concave lens out. If you do not see any image on the screen (arrows have to be disappeared.), that is a correct place where you put the concave lens. If it is still a blurred image, it is a wrong place for the concave lens to give you the final focused image.

Lens separation $r$ : $\qquad$ Object distance $s_{2}=r-s_{1}^{\prime}$ $\qquad$
Image distance $s_{2}^{\prime}$ : $\qquad$
Focal length of diverging lens, $f=s_{2} s^{\prime}{ }_{2} /\left(s_{2}+s_{2}^{\prime}\right)$ : $\qquad$ ( $\sim-20.0 \mathrm{~cm}$ )

- To confirm the results, calculate the focal length of the diverging (concave) lens and the total magnification.
$*$ Magnification of the diverging lens
$M_{2}=-s_{2} / s_{2}:$ $\qquad$ (only from the distance ratio)
*The total magnification
Direct measurement $\rightarrow M=\frac{\text { final image height, } h_{2}^{\prime}}{\text { original object height, } h_{1}}$ $\qquad$
Distance ratio $\rightarrow\left(s_{1}^{\prime} / s_{1}\right)\left(s_{2}^{\prime} / s_{2}\right):$ $\qquad$


## Questions:

- Did you get the experimental focal length for the diverging lens close to the reference value ( 20.0 cm or 0.200 m )?
$\square$
- For the third part, is the measurement of magnification consistent? (The direct measurements and distance ratios are supposed to be the same.)
$\square$
- After inserting the diverging (concave) lens, is the previous image magnified or reduced?
$\square$


## Questions you want to explore:

In the second part of this lab, the total magnification with two lenses is enlarged. Namely, a small object is magnified as the image. The combination of convex and concave lenses is applied to the Galileo telescope. Let's think of the following:


