

# Vector Addition and Physical Significance

TA's signature allowing you  
to take the quiz or leave \_\_\_\_\_

Your Name \_\_\_\_\_

Partners' Names \_\_\_\_\_

Obtained reasonable experimental results?	yes	<input type="checkbox"/>
Answered questions?	yes	<input type="checkbox"/>
Cleaned your table?	yes	<input type="checkbox"/>

## Introduction

Numbers can be added or subtracted. However, some physical quantities cannot be calculated by such way. Let us think about pushing a car by two people. Suppose they are able to push it with 700 newtons. (Newton is a unit of force.) Can they always push the car together with simply the sum of their forces,  $700 + 700$  newtons? The answer is: It is not always true that they can. If they push the car with completely opposite directions, the car will hardly move. Directions play a large role in this situation. Such value containing both magnitude and direction is called vector. Some of the physical quantities are vectors and it can deal with the calculations more easily. Displacement, velocity, and acceleration are vectors in this sense. Some physical quantities, which refer to only magnitude are called scalar. Some of the scalars are named differently, such as distance and speed although the vectors are called displacement and velocity. If there are two vectors whose magnitudes and directions are equal, they are identical vectors even though they are located at different place. To calculate vector addition quantitatively, it is convenient to obtain each component from the coordinate system, yet vector addition can also be dealt with graphically. Another important concept is the equilibrium of multiple vectors. If they are forces, the object will not move or will stay at a reference point. Thus, the sum of these vectors gives zero.

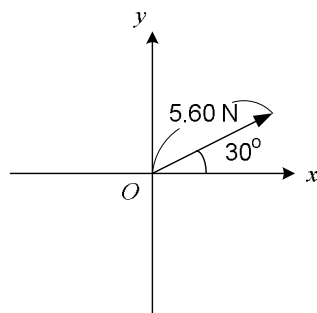
### Objectives:

- To learn physical and mathematical properties of vector
- To learn how to add vectors
- To verify that a static equilibrium of vectors results in zero as the sum of these vectors

## 1. The x- and y-components of vectors

In a two-dimensional space, a vector can be expressed by its x- and y-components. **The following method (angle taken from positive x-axis) is systematic; then, you can use cos and sin for x- and y-components always, respectively.**

e.g.

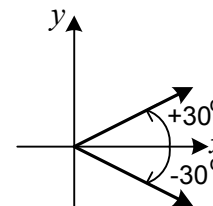


The vector has a magnitude, 5.60 N, and is directed  $30^\circ$  from the positive x-axis. Therefore, each component of the vector will be:

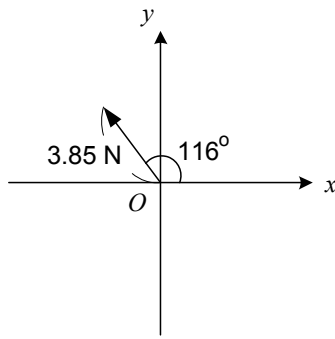
$$x\text{-component: } 5.60 \text{ N} \times \cos 30^\circ = \underline{4.85 \text{ N}}$$

$$y\text{-component: } 5.60 \text{ N} \times \sin 30^\circ = \underline{2.80 \text{ N}}$$

**Note:** The angle taken from the positive x-axis counterclockwise is positive, but when it is taken clockwise, it will be negative angle. Also, for sin or cos functions,  $-30^\circ$  gives an equal result with  $+330^\circ$ .



Obtain the components for the following cases:

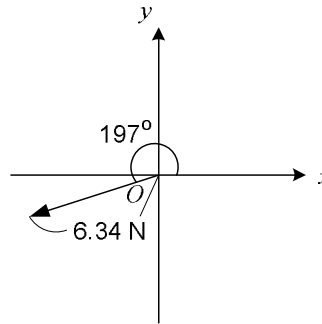


x-component = \_\_\_\_\_

y-component = \_\_\_\_\_

x-component = \_\_\_\_\_

y-component = \_\_\_\_\_

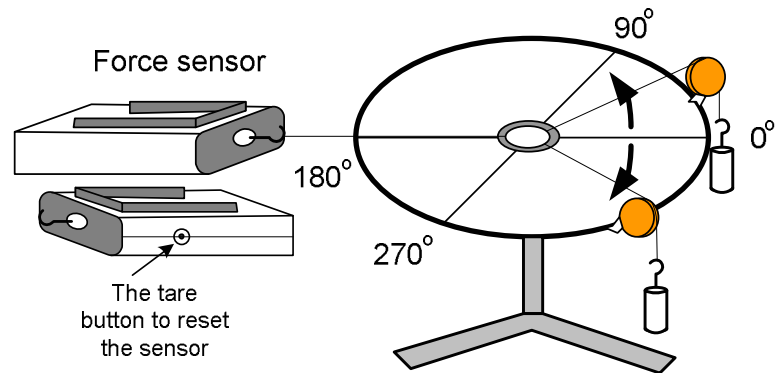


**Question:**

Describe a general procedure to obtain the x- and y-components of a vector. Where do you take the angle? When do you use sine or cosine? How do you explain this to your friends?

**2. Significance of using vectors**

- ① Set up the following experiment.
- ② Use the same mass for the hanging masses, say, 200-g mass for each.

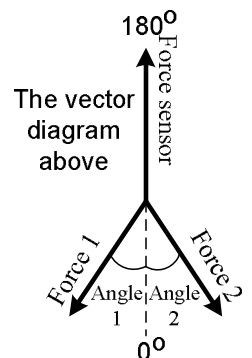


\* Start up DataStudio. Click “**Create Experiment.**” Click an **analog channel** of the interface and select **force sensor**. Look in the “display” box (left side of the window) and select “3.14 Digits.”

- ③ Follow the data table and the figure to make the angles. The angles have to be taken symmetrically (angle 1 = angle 2). Record this in the table below.  
[First, reset the force sensor by pressing the tare button without any mass before starting this experiment.]

- ④ Increase the angles as instructed; calculate the force (weight); and then read the net force from the force sensor by following the data table below.

**Note: The force due to gravity is that mass × g (9.8 m/s<sup>2</sup>) and its unit is N.**



1		2		3	4	5	6
Hanging masses (Use the same mass for both hanging masses for all of the trials.)		Forces due to gravity (N) (Multiply 9.8 m/s <sup>2</sup> by the mass in column 1.)		Force 1 + Force 2 (N)	Angle 1 and Angle 2 (degrees) (Change angle 1 and angle 2 as shown in the figure above.)	The net force with force sensor (N) (Read the force sensor. Negative sign indicates a pulling force.)	Force1×cos(angle1) + Force2×cos(angle2) (N) (Use columns 2 and 4.)
Mass 1	Mass 2	Force1	Force2				
0.200 kg	0.200 kg				0°		
0.200 kg	0.200 kg				10°		
0.200 kg	0.200 kg				45°		
0.200 kg	0.200 kg				90°		

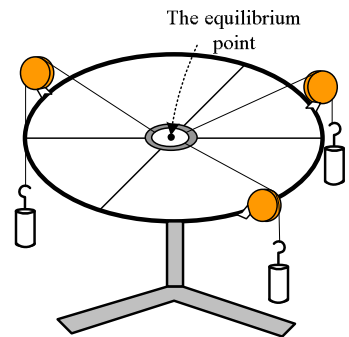
**Question:**

The magnitude of two forces by the hanging masses is unchanged. Thus, the force sensor could detect the same force every time. However, by changing the angles of the forces, the net force varies as you have seen in the above result. (Compare columns 3 with 5.) Explain why this happens and describe importance of the consideration of the vector concept. (Imagine when you and your friend pull a car as above.) Then, what is column 6? Why are they close to column 5?

**3. Static equilibrium with three forces**

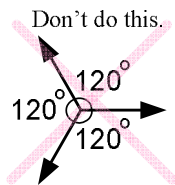
Procedure:

- ① Remove the force sensor.
- ② Use 3 different hanging masses as shown in the figure. (These are three vectors.)
- ③ To make the equilibrium of three force vectors, adjust the angles so the center ring can be located in the middle of the nail.
- ④ Calculate each force and the net force by following the table below.



① Use different masses and angles for each hanging mass.

	Mass (kg)	Force (=9.8 m/s <sup>2</sup> ×Mass)	Angle	F <sub>x</sub> (Go back to the first part.)	F <sub>y</sub> (Go back to the first part.)
Force 1					
Force 2					
Force 3					
Sum of x- and y- components of each force ⇒					

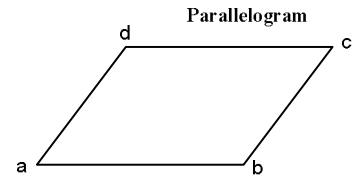


**Question:**

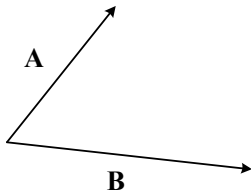
Do the force components add up to zero? (You are supposed to obtain zero theoretically.)

## How to add vectors graphically

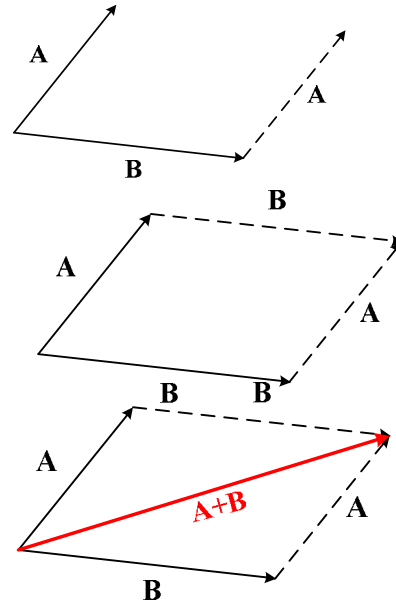
Definition of parallelogram is that side  $\overline{ab}$  must be parallel with side  $\overline{dc}$  and the length has to also be equal. Likewise, sides  $\overline{ad}$  and  $\overline{bc}$  are equal and parallel.



Two vectors can make a parallelogram as follows:



Vector A can be moved as shown.  $\Rightarrow$



Likewise, vector B can be replaced as shown. All of these construct a parallelogram.

The addition of vectors A and B can be found as follows. It has to be drawn from the start point of the vectors so it can be the diagonal of the parallelogram.



For the last part of this lab, read the following instruction. Use the above data and add all of the vectors graphically.

**The data shown below are only for the explanation purpose. Please do not use this for your own results.**

- After taking data, you have the information of all of the vectors (the magnitudes and directions). The force column corresponds to the magnitude; and the angle column corresponds to the direction.

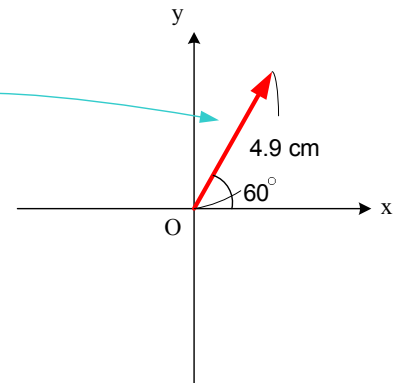
### 1. Static equilibrium with three forces

	Mass (kg)	Force (=9.8m/s <sup>2</sup> ×Mass)	Angle
Force 1	0.50 kg	4.90 N	60°
Force 2	0.50 kg	4.90 N	180°
Force 3	0.50 kg	4.90 N	300°
Sum of x- and y- components of each force $\Rightarrow$			

- Draw each force vector as follows: Force 1 has magnitude, 4.9 N, and direction 60°. For convenience' sake, draw a vector on paper like 4.9 cm.

### 1. Static equilibrium with three forces

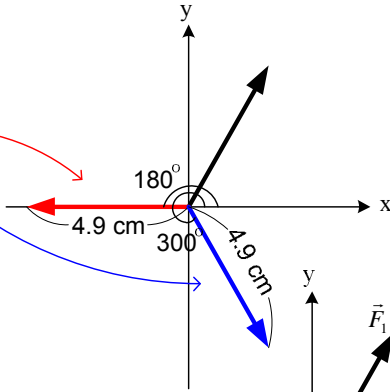
	Mass (kg)	Force (=9.8m/s <sup>2</sup> ×Mass)	Angle
Force 1	0.50 kg	4.90 N	60°
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Sum of x- and y- components of each force $\Rightarrow$			



1. Static equilibrium with three forces

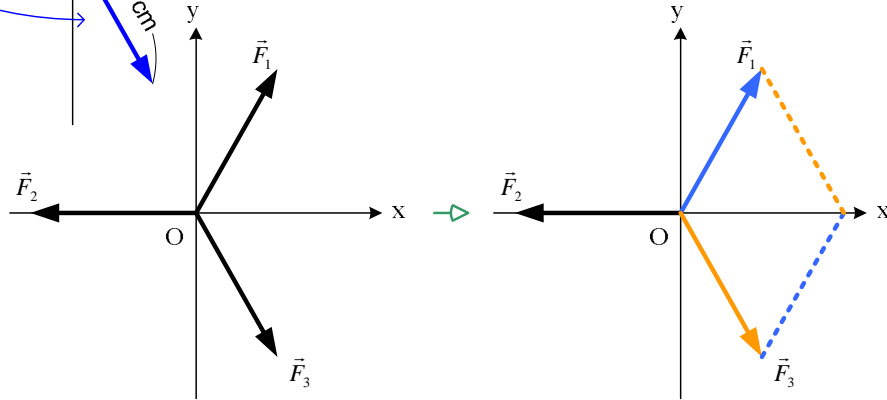
	Mass (kg)	Force ( $=9.8\text{m/s}^2 \times \text{Mass}$ )	Angle
Force 1	0.50 kg	4.90 N	$60^\circ$
Force 2	0.50 kg	4.90 N	$180^\circ$
Force 3	0.50 kg	4.90 N	$300^\circ$

Sum of x- and y- components of each force  $\Rightarrow$

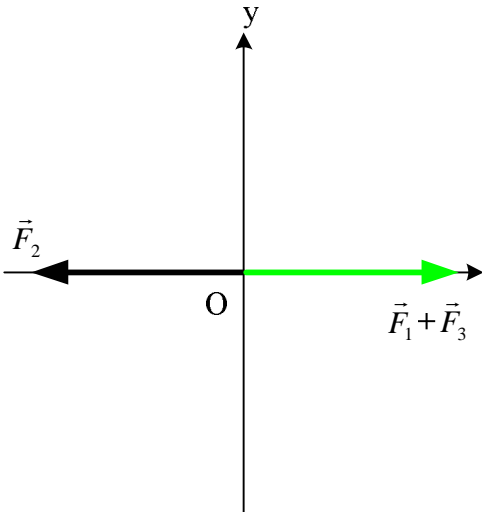
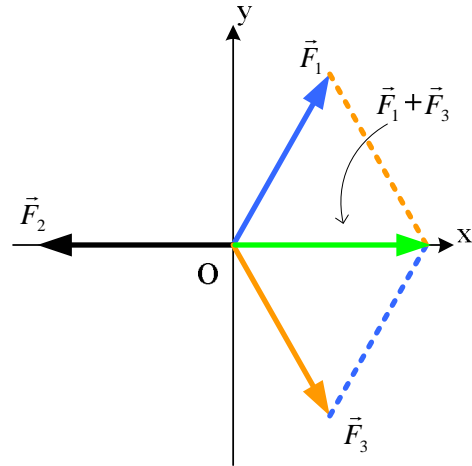


3. Perform the same procedure for the second and third vectors. The angles are always taken from the positive side of x-axis.

4. After you draw all of the vectors, pick out two vectors. Then, make a parallelogram as shown.



5. The diagonal will be the sum of two vectors. (Be careful about how you take the diagonal. The starting point of the vector is the origin.)



6. Two vectors added becomes one vector as shown. Repeat the same procedure as above until you obtain one final vector. Now, there are two vectors, which are  $\vec{F}_1 + \vec{F}_3$  and  $\vec{F}_2$ . If either one of them is tilted, you can make a parallelogram. However, in this example, these vectors are an equal magnitude and opposite directions. Therefore, the sum of two vectors has cancelled each other.

7. The sum of three vectors becomes a zero vector as shown. However, practically, considering the experimental uncertainty, you likely obtain a small vector from origin as the error.

